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Mathematics in American Society 1888–1988
A Historical Commentary

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1. THE PLACE OF MATHEMATICS IN AMERICAN SOCIETY

Two histories. For the publication of new mathematics by American mathematicians, the century that began in 1888 has been one of promise and fulfillment. It is appropriate to celebrate that achievement on the occasion of the centennial of the American Mathematical Society.

Then there has been another history: the story of how mathematicians have practiced their profession and earned their living in American society, and of how that society has treated its mathematicians. It is a story of research, scholarship, and teaching, of the practice of mathematics in science, industry, and government, of industry's use of mathematics, of society's support through education and grants of money, of mathematicians as citizens and citizens as mathematicians. For this history 1888 was also a time when mathematics had high status and great promise. But thereafter both status and position have eroded, except for a brief heyday from about 1950 to 1970. So, as a contribution to a complete story of American mathematics, let us

proceed to a brief review of some aspects of this second history, the social history of American mathematics.

Institutions that include mathematics. In American society higher education and secondary education are two of those institutions that are allowed to govern their territory so long as they obey the law and stay within the budget. Mathematics itself is not such an institution, but it belongs to both higher education and secondary education. Much trouble, particularly in mathematics, has resulted from the separate, and almost independent, governance of these two institutions. However, the histories of these two institutions have been written; and they provide the best available framework for the social history of American mathematics, [1–7]. To a remarkable extent they fill in the gaps in the social history of mathematics, which is only sketchily recorded, or the history of a single mathematics department [8].

Mathematics also belongs to less well-defined national institutions called science and technology whose official interface with the federal government is the National Academy of Sciences–National Research Council (NAS–NRC). In practice this is the main interface of mathematics with the government. In the government, we have the National Science Foundation (NSF) and the National Institute of Health (NIH), while in the research community we belong to the American Association for the Advancement of Science (AAAS).

In these scientific institutions we again have special difficulties. For mathematics is not entirely a science, having a humanistic culture as well which resides more comfortably in higher education. Moreover, among the sciences, our basic research tends to have a long delay before providing visible social utility. And our applied mathematics tends to be a tool subject, a problem-solving technology for problems that arise outside of mathematics. Either way we seem to fit in more as "servant of the sciences" than as a science in our own right. Our own associations, AMS, MAA, SIAM, ORSA, NCTM, and the statistical associations, have no direct interface with the government.

2. UNLIMITED PROSPECT

Mathematics in the general studies curriculum. In 1888 mathematics had a secure place in American society [9]. This was based on its long-established position, along with the Latin and Greek classics, as one of the required general studies that had made up the curriculum since the earliest medieval universities [11,13]. It was one of the essential humanities. In 1888 Daniel Gilman, in his thirteenth report as president of the new research university, Johns Hopkins, said with justifiable hubris: "How to begin a university... Enlist a great mathematician and a distinguished Grecian; your problem will be solved... Other teachers will follow them..." [14, p. 29]. Gilman's "great mathematician" was J. J. Sylvester and his "distinguished Grecian" was the renowned Gildersleeve.

Secondary school mathematics requirement. The position of mathematics was further bolstered by its requirement in the secondary school curriculum. There were some public "academies" preparing students for college but in general this was the function of independent private academies. Although there was no consistent standard, their curricula were designed to prepare for the general studies curriculum of the colleges by a required general studies curriculum of their own, dominated by the entrance requirements of the colleges. Mathematics was required for graduation from the academy even though the substance of this requirement was somewhat ragged.

The great boom. The period after the American Civil War up to 1893 saw the greatest boom in higher education in the nation's history. New state universities were established, including the new land grant colleges, such as MIT and Purdue, that included schools of engineering and applied science. There were new colleges for women and for blacks. During the war Yale had taken the lead to establish a Ph.D. program; and more such German-style graduate degree programs followed. Science had finally made its way into the university curriculum over the opposition of the entrenched humanities, although in a separate B.S. degree program segregated from the traditional liberal arts. Hence the name of the central university college was changed from "the academical college" to "the college of arts and sciences." And the national learned societies came into being, including the AMS in 1888.

The position of mathematics was further fortified by its requirement in the curricula of the new B.S. degree programs and those of the new schools of engineering and applied science.

European resources. American mathematics still drew strength from European and English mathematics. Young Americans with ambitions for mathematical research went to Germany, particularly Göttingen, for their Ph.D. degrees. A variation of this pattern was emerging in which one got the Ph.D. in one of the new American graduate schools, and then went to Germany for a year or two of further research training. English or French mathematics texts were still the mainstay of instruction in both secondary school and the university. (The Germans wrote treatises, not textbooks.) Distinguished European mathematicians were still being attracted to bolster the faculties of the new American graduate departments.

Applied mathematics. Applied mathematics appeared to be in good shape in 1888. True, the old applied mathematics consisting of the application of trigonometry to surveying, mapping, and military science had run its course and was being taken over by civil engineering. But mathematics then embraced rational mechanics and positional astronomy. This was the applied mathematics of 1888. Only a few American mathematicians were doing significant research in celestial mechanics, but they taught the classical work of Newton, Lagrange, Laplace, and Gauss. Of course there was the brilliant

and original work of Willard Gibbs in phase equilibria, chemical thermody-
namics, and statistical mechanics. He had, however, little in the way of a
research group to continue and develop stochastic applications of mathemat-
ics. In fact, the whole field of rational mechanics had become too difficult
technically for all but a very few graduate students in the new doctoral pro-
grams. They were not prepared to work in the statistical mechanics of Gibbs
or the new topological dynamics of Poincaré.

3. THINGS GO WRONG FOR AMERICAN MATHEMATICS
1893–1940

Retrenchment. We may date the end of the great boom in higher education
as 1893, the year of a severe national financial panic in the second adminis-
tration of President Cleveland. As we shall see, there were also other reasons,
more specific to mathematics, for regarding this as the end of an era and the
beginning of another. The years 1893–1940 saw a slow but substantial growth
in American mathematical research. In this respect it was a period of great
achievement. But for the position of mathematics in American society the
unlimited prospect of 1888 was not to be realized. Very shortly after 1888 a
series of disasters fell on mathematical education, on applied mathematics,
and on the service of mathematics in American society.

Mathematics loses its traditional place in the required general studies. The
traditional curriculum of required general studies had been carried over into
the expanded colleges from the early colleges that trained students for the
ministry, or for teaching. That difficult course of studies included Greek
and Latin for their cultural values as well as for reading the scriptures in the
original. It included mathematics, partly by sheer weight of tradition, and
partly for "training the mind." By 1888 it was apparent that there would
never be enough jobs as minister of the gospel, or teacher, to employ the
flood of B.A.s then graduating. Nor were there jobs enough in the technically
primitive industry to employ the graduates of the new science and engineering
programs. And students in the traditional degree program were clamoring
for a German-style free elective system to replace the difficult and outmoded
general studies curriculum.

Charles W. Eliot, President of Harvard (1869–1909), had started his ca-
reer as a mathematics tutor (1854–1863), before changing to chemistry and
then to the presidency of Harvard. He was undoubtedly the most influential
educator of the century [15]. He took the lead to give the students what they
wanted, a free elective system. Other universities quickly followed Harvard's
lead. It was the ruination of classics, replaced by a newly synthesized subject
called "English." Mathematics lost a large share of its enrollment and could
have suffered the same fate as classics if it had not been for its requirement

in the new physical sciences and engineering. Also there was no popular substitute for mathematics, no computer science! Thereafter mathematics was even more strongly regarded in the national government as a subsidiary of the physical sciences and engineering.

The wide adoption of the free elective system created so much chaos in the college curriculum that some constraints had to be reintroduced to restore order. One of these was the requirement of a major subject, introduced by Woodrow Wilson as president of Princeton. Both Harvard and Yale also introduced systems of group requirements and academic advising that, together with the major, make up the system we have today. But this did not restore the old position of mathematics.

Mathematics loses out in secondary school. A major trouble with the great expansion of higher education that had occurred was that it was built on an inadequate foundation. There was no national system of secondary schools to prepare students for the new public universities. A blue ribbon "Committee of Ten", headed by none other than Harvard's Eliot, was commissioned to decide what the new schools would be. In a report in 1893, it rejected the English, French, and German models. The committee recommended a new American high school that would be under the same local school boards that controlled the universal public elementary education. The additional four years would be offered, but not required. The colleges, which had dominated the old academies that prepared students for college, were to have no authority over the new high schools. It was believed that the majority of the new students would not go to college but would only add four years to their terminal general education. Many would profit by vocational, rather than academic, education [16].

While the new high schools were to offer college preparatory mathematics courses, these courses were not built into the graduation requirements as they had been in the academies. The estimates at the time called for only 10 or 15 percent of public high school students to take the college-bound curriculum.

These arrangements left the school-college interface in some confusion, so a new Commission on College Entrance Requirements in 1899 defined the boundary and set up the system of Carnegie units. These describe the content of a student's record. The minimal college entrance mathematics content was 2.5 units, including one unit of deductive Euclidean geometry. In addition, the report called for the establishment of the College Entrance Board (CEEB) which would prepare and administer standard examinations to validate the graduation credits. This strengthened the position of the private universities that required the CEEB scores. Originally the state universities, and many private ones, did not require the CEEB scores but the system of Carnegie units was universally used [17,18].

These arrangements gave the major private universities at least the same control of their admissions that they had enjoyed before 1893. In subsequent years these universities would exert the only real force for maintaining quality of performance in secondary schools. In the state and private universities that admitted on a basis of the high school diploma, there was little control of admission standards. Of course this affected mathematics most of all. The mathematicians hated it, but there was little they could do about it. Moreover, at first it was true that the new public high schools were supplying only a minor fraction of college admissions; so the issue was slow in coming to a head.

PEB. James B. Conant [19] has praised the American high school for its democracy and adaptability to local conditions. As parents and citizens we can all appreciate the undeniable virtues. But it has not worked well for mathematics in American society. We recognize that there are other important educational and social objectives besides the teaching of mathematics. But we, as custodians of the mathematical culture, have not only the right but the responsibility to look after the quality of mathematics that the secondary schools provide. We have already seen one way in which the system was flawed for providing good mathematics. It was seriously flawed in yet another way.

The local school boards had no authority, or capability, to educate or certify teachers, to establish a national curriculum, or to acquire and distribute state and federal funds for education. The federal government was deliberately denied such powers. And the universities did not have these powers or responsibilities. Somebody had to run the public school system; Eliot's plan did not provide for it. In this power vacuum there arose an unofficial Professional Education Bureaucracy (PEB). PEB came to consist of the superintentents who act for the local school boards, the state and federal officials that control and distribute the public funds and certify teachers, and the deans of the schools of education that train teachers. The U.S. Commissioner (now Secretary) of Education has never been a member of PEB. He is a temporary federal appointee of the president, who is permitted to sound off somewhat impotently. Unofficial PEB has never had an official organization. It speaks, when necessary, through some committee of the National Education Association (NEA) as Eliot's Committee of Ten did. The leaders of PEB occupy politically vulnerable positions, especially the superintendents, who are the key members. But the actions of PEB, in setting the national curriculum, in training and certifying teachers, and in distributing state and federal funds, are not directly subject to democratic approval. In particular, they are insulated from the criticism of the intellectual, university, and scientific communities.

In 1913 NEA appointed the Commission on the Reorganization of Secondary Education. It clearly reflected the thesis of John Dewey that the function of the school is to foster the growth of the pupils along the lines of their own interests rather than to impart subjects, and that the aim of education is social. This philosophy avoided the European objectives of liberal education and culture. But the Commission consisted of 14 committees representing subjects. Only mathematics was not represented. The general report, issued in 1918, set forth the agenda of social development and personal fulfillment as the aims of secondary education, and relegated the mastery of subjects to low priority. These objectives gained wide acceptance without any formal adoption into national policy [20].

Isolation from European Mathematics. Not only was the new American high school different from its European counterpart; the implication was that the American college program would be different too. European textbooks, course structures, and teachers were no longer transferrable to America as they had been in the past. The first result was that American mathematicians had to write new textbooks to fit the new courses. The last of the French textbooks to be translated and widely used in this country was that of Goursat and Hedrick (1904).

One of the first significant new American texts was Granville's *Calculus* (1904). Since college algebra was a new course, textbooks had to be written for it. The subject was a kind of hodgepodge. About half of it had to be a review of high school mathematics. This was in effect remedial mathematics from the start. The rest of it advanced into the study of polynomial algebra, including Horner's method for the numerical solution of equations of any degree. H. B. Fine of Princeton and Wylczinski and Slaught of Chicago wrote significant textbooks that were considerably higher in level than later ones. Fine also wrote a distinguished analytic geometry textbook. Since synthetic Euclidean geometry was assigned to the high school, this course was geometry on its own merits, continuing Euclidean geometry by Cartesian methods, not just precalculus coordinate techniques. L. E. Dickson of Chicago wrote a junior level polynomial algebra text, called *Theory of Equations.* Like college algebra, the analysis to follow calculus was something of a medley, called advanced calculus. E. B. Wilson of Yale wrote one of the significant texts.

The effect of all this writing of American texts was to divert energies from research to teaching, and to isolate American from European mathematics. Graduate work also turned inward. Not so many Americans went to Europe, even for postdoctoral study. The faculties of colleges and state universities were now American educated. Moreover, since so much low-level and remedial teaching was required, the Ph.D. degree was not considered necessary for all the faculty. A number of the universities that had previously announced Ph.D. programs quietly let them become inoperative and became large undergraduate colleges with only a Master's program attached. Research was

concentrated in relatively few universities; and even they had less contact with Europe than they had earlier.

American mathematics loses relevance in science and technology. Not long after 1888 American mathematics lost mechanics, which had been its applied field and its connection with science and technology. In part this was due to the new American research interests in such subjects as algebra and topology, which could be undertaken without so much technical background as mechanics had come to require. In part it was due to the collapse of the Newtonian program to construct an axiomatic mechanics as an extension of Euclidean geometry. Maxwell's electromagnetic field physics had put this out of reach, and the emergence of Einstein's relativity and the new quantum theory of Planck clinched it. Mach's *Science of Mechanics* [21] was a historical farewell to the Newtonian program. The teaching of mechanics was gradually taken over by physicists and engineers, who trashed it!

Research in astronomy, too, moved toward physics, that is, into the analysis of the spectral quality of light gathered by large reflector telescopes. It was called astrophysics. Astronomy departments divorced themselves from mathematics and taught the celestial mechanics themselves as part of their continuing, but secondary, interest in positional astronomy. Gibbs's statistical mechanics continued to be studied mainly in physical chemistry and thermodynamics. That left the continuation of Poincaré's program in topological dynamics, which was taken up by G. D. Birkhoff and a few of his students, as about the only part of mechanics left to mathematics. And at the time that had little contact with the physical sciences. All told, that left American mathematical research and advanced teaching with virtually no viable applications in science and technology.

Nobody realized what was happening. E. H. Moore, in his 1902 retiring address as president of the AMS, entitled "*On the foundations of mathematics*" [22], gave a remarkable summary of the state of American mathematics and mathematical education at the time. He did not see what was happening. He saw only the continuation of the old relationship to European mathematics and the continued leadership of lower school mathematics by the university mathematicians. And he was optimistic about American applied mathematics. He did not realize the consequences of the loss of mathematics's traditional requirement in the general studies curriculum. In fact nobody seems to have realized what was happening.

One of the first to speak about the parallel between the demise of classics and the near demise of mathematics was E. B. Wilson in 1913 [23]. He wrote:

> The decline of the Graeco-Roman empire over our collegiate studies is apparently extending almost to extinction. That culture and valuable intellectual discipline are best obtained by application to subjects which are neither useful nor interesting to the student,

and over which he never obtains even a mediocre mastery, is an idea which is losing ground despite the entrenchment of vested interests. The fact is that Greek and Latin do not make good.

We mathematicians, however, are in no position to gaze upon the motes in the eyes of our classical bretheren, to whom we can hardly compare ourselves favorably. For there has been a great decline in the sway of mathematics over our collegiate studies. We suffer by the presence in college of great numbers of fellows neither primarily nor seriously there for the sake of intellectual advancement. But our chief difficulty is that we do not make good.

Wilson went on to propose that college mathematics should begin with "early calculus" for its intellectual value and real applications. Elsewhere he modified this by saying that first-year mathematics should consist of equal amounts of calculus and "choice and chance." By the latter he meant something like what we would call finite or discrete mathematics, with an introduction to probability and its applications. If we may look ahead in this history, we must report that Wilson's proposal did not take hold very widely, although Harvard took the lead somewhat later to establish calculus as its standard freshman course [24]. And in the years after 1953 the MAA Committee on the Undergraduate Program in Mathematics (CUPM) undertook to install Wilson's program of calculus and choice and chance as the standard first year mathematics in American colleges. After some early success, that initiative was overwhelmed by the tide of remedial, precalculus mathematics but left a substantially wider adoption of calculus and elementary combinatorics as the normal first-year mathematics. But, going back to 1913, it is doubtful that Wilson's proposal would have satisfied the engineers; for at that time they used finite difference and sum methods that they did not recognize as calculus.

By 1916 the mathematicians became alarmed at what was happening in the NEA Commission on the Reorganization of Secondary Education, in which mathematics was not represented. Belatedly E. R. Hedrick, as first president of MAA, appointed a national committee to prepare a statement for mathematics. It had representation from the various mathematical and educational organizations involved in high school mathematics. It was called "The National Committee on Mathematical Requirements." It was chaired by J. W. Young of Dartmouth, and it included E. H. Moore, Oswald Veblen, and D. E. Smith, as well as similarly prominent teachers and administrators from the secondary school system. But World War I was on, and the committee did not report until after the war ended. Meanwhile other events intervened to aggravate the problem.

Expansion strikes again. Before America's involvement in World War I, a national agreement was reached to extend required public education from

8 years to 12. Previously the high school component was provided but not required [25]. When the plan was implemented after the war, the sudden demand for teachers could not be met while maintaining quality standards; and if the students had been deficient before, these compulsory registrants were worse [26]. There was near chaos. Soon these students swarmed into the colleges and universities. Administrators were eager to have them for the growth they propelled. The mathematicians, with their outmoded standards, were blocking progress. The attack on mathematics requirements intensified. PEB was now much stronger politically. The education schools to train teachers were well established, if strained to meet the demand. PEB was joined by the college deans, many of whom now came from the popular social sciences where they had little use of, or appreciation for, mathematics. But, in spite of these difficulties, the expansion provided an opportunity to establish new graduate programs in mathematics. So the expansion produced a lot of new Ph.D.s in mathematics, too. They oversupplied the weak demand for college mathematics teachers; hence salaries were low. Then came the Great Depression in 1932. All academic professionals were hit, but mathematicians fared especially badly because of the oversupply. It was the worst of times for American mathematicians.

The Committee on Mathematical Requirements. We return to the J. W. Young committee, which Hedrick had appointed in 1916. In 1923 it finally issued its 652 page report that was probably the most thorough study of school–college mathematical education ever done. It titled its report, "The Reorganization of Mathematics in Secondary Education," presumably to identify it as the mathematics statement for the 1918 report of the NEA Commission on the Reorganization of Secondary Education [27]. The Young committee also made an extensive survey of secondary school mathematics curricula and training of teachers in foreign countries, which showed the United States to be deficient. It examined many available national tests of ability and achievement in mathematics. It presented sample units of exposition for topics of critical difficulty for the student. In particular it offered an outline of an intuitive approach to plane geometry for weaker students while retaining the traditional deductive course for college-bound students. It stressed the importance of secondary school mathematics for science and industry in this country.

It is fair to say that these authoritative and thoughtful recommendations came to nothing. By this time PEB had powerful political influence and the mathematicians had virtually none [28]. Moreover mathematics had few allies in the universities. And there seems to be no record of its seeking support from the National Research Council.

The tide begins to turn for American mathematicians. It was 1940 before the American public became aware again that mathematics is important, but years earlier some things happened in mathematics to reveal that recovery

was under way. Recovery from irrelevance came first. Significant new fields of applications of serious mathematics opened up, though the public was not yet aware of this. (We will look at these new applications in the account of the postwar period as they unfolded and came into use in American society.)

Another great new start occurred when the Bamberger–Fuld families made a major gift to establish a new research institute above the level of the existing graduate schools. They entrusted the development of the idea to Abraham Flexner [29]. His idea was to choose, as the initial field, one in which there would be universal public recognition of the eminence of the chosen scholars and their work. Consultation with intellectual leaders in this country and in Europe convinced him that that field was old mathematics, apparently somewhat to his surprise. So the Institute for Advanced Study was established in Princeton in 1930 with mathematics as its first field of study and with Einstein as its first "mathematician" [30]. Veblen came over from Princeton as its first chairman [31]. Einstein suggested von Neumann. Herman Weyl came. Kurt Gödel came as Einstein's assistant. And the Institute for Advanced Study was under way.

The distinguished European mathematicians who joined the new Institute were only the first of a wave of fine European mathematicians who soon followed, fleeing Hitler and oppression. But they came at the very depth of the Great Depression, a time when many American mathematicians had no jobs, or poorly paid teaching jobs. There really was not room for the Europeans. But Veblen and AMS Secretary, R. G. D. Richardson, appealed to mathematics departments in the United States and Canada to find places for them. Veblen assigned to Leon Cohen the task of raising funds by contributions from American individuals and businesses to subsidize new jobs for a list of fine European mathematicians prepared by Veblen. Somehow positions were found for them in the United States and Canada. These immigrant mathematicians not only broke the isolation of American mathematics from that of Europe, they became the leaders in research of a great new era of American mathematics [32]. It was more than a triumph for mathematical research in America. It was a great day in the social history of American mathematics as well [94–97].

In 1938, the semicentennial year of the AMS, R. C. Archibald's history of the AMS revealed little about the status and service of mathematics in American society except that the Society was growing and gaining support after difficult times [84].

But G. D. Birkhoff's address, although primarily a survey of American achievements in research, also spoke with Birkhoffian bluntness about the status of American mathematicians. He quoted a prewar survey by J. C. Fields of Toronto which concluded that among mathematicians throughout the world "the American professor is the worst treated of all." Birkhoff gave reason to think that, since the war, things had improved. "There are now

many chairs where the salary is good and the duties not excessive." On the other hand he worried that so many of those chairs went to distinguished Europeans, leaving capable Americans "as hewers of wood and drawers of water," adding, "it should be strongly emphasized that twelve hours of instruction each week is about all that can be required if the best standards of scholarship are expected." Then, expressing gratitude to the public for having supported mathematics so generously "if unwittingly," he added: "On our part there is an inescapable deep responsibility to the nation. It is our duty to take an active and thoughtful part in elementary mathematics instruction as well as to participate in the higher phases" [85]. He meant it. The Birkhoff–Beatley textbook for a simplified, but still rigorous, school geometry was one of his contributions.

4. RENAISSANCE 1940–1970

War makes mathematics important again. The approach of war in 1940 changed the climate for American universities. Now, for the first time in many years, a male student had to compete for admission on a basis of his performance in science and mathematics. A good performance was rewarded by a delay in military service and a college education at government expense. That would qualify him for a commission in the Army or Navy when his time came to go to war.

Then, in the war, it turned out that science, physics especially, made major contributions to the military victory through such innovations as radar, sonar, gun- and bomb-guiding computers, and finally the atom bomb. But mathematics was involved both in the original design and development of those weapons and in their most effective use [33–34]. To cite one outstanding example, the statistician Sam Wilks and the physicist Philip Morse made daily best estimates of the position of the German submarine fleet on the basis of the previous day's sightings. The superiority of sophisticated mathematical analysis over guessing in such battle situations resulted in the attachment of operations research, or operations analysis, sections to the military commands [35]. Moreover, the great importance of a large, high speed, electronic computing machine was recognized for such applications as logistics and secret intelligence, as well as the design and development of new weapons [36].

Basis of the return from irrelevance. While the war brought mathematics to the public consciousness again, the basis of the renewed usefulness of advanced mathematics in American science, technology, and organization had been established earlier, unknown to the public. It was not a product of the war, and it continued to develop with renewed impetus after the war. One significant start was the discovery that complex analysis was essential to the new electronic engineering that developed out of radio in the years between

1900 and 1925. In particular, by that time the Laplace or Fourier transform was recognized as the basic mathematical tool for the theory. Another, entirely different, kind of mathematics found new uses in electrical engineering. It was popularly presented in T. C. Fry's *Probability and its Engineering Uses* (1928).

The mathematical basis of the new applications of probability was set up by uniting the statistical methods of R. A. Fisher (1932) and von Mises (1939) with Kolmogorov's measure–theoretic formulation of mathematical probability (1933) [37]. Influential first expositions of the new mathematical statistics that resulted from this union were Cramér (Princeton, 1945) and Feller (1950) [37]. From these roots mathematical statistics grew into an enormously powerful and pervasive technology.

The completion in the years 1925–1930 of the linear operator formulation of quantum mechanics opened the door to a vast array of applications in physics, chemistry, solid state engineering, and, surprisingly, biology. The full implementation of this revolutionary development had to await the availability of the large-scale computer after 1945 [36]. It led Weyl to remark: "... this branch of mathematics [linear algebra and analysis] crops up everywhere in mathematics and physics, and knowledge of it should be as widely disseminated as the elements of differential calculus" [38].

The success of operations analysis in World War II gave a great impetus to the mathematics of optimization: operations research, linear programming, game theory, input–output analysis, optimal control, stochastic control, etc. These mathematical methods found more and more users in businesss management.

It was not only branches of mathematics, finding new applications, that reversed the inward turning of American mathematics that had isolated it from science and society for 50 years; it was also mathematicians and mathematical scientists, many of them the European emigrés. There was Einstein. General relativity at first had little impact on society. Among intellectuals it was either a humorous or outrageous paradox. Among professionals in mechanics it was a complicated theory that made small corrections in, still valid, Newtonian mechanics. But the atom bomb in 1945 and the subsequent nuclear energy technology changed all that. Even the common man now recognized relativity's mass–energy equivalence, $E = mc^2$. Moreover, in the modern science of the cosmos, popularized on TV, it has turned out that relativity is the only valid mechanics [86].

And there was von Neumann. His theory of games won almost instant recognition in economic theory and business technology. But his 1948 paper, "The General and Logical Theory of Automata", has had a slowly developing, deep impact on both computer architecture and biological science [87–88].

Other mathematical scientists of the years before and after World War II who brought serious mathematics into new uses in science and technology included Hermann Weyl, Vannevar Bush, Norbert Wiener, Claude Shannon, Philip Morse, Kenneth Arrow, Wassily Leontief, John Kemeny, and others. The list is uneven in terms of magnitude of mathematical content; we are talking about mathematical scientists whose work was influential outside the mathematical community.

And there appeared on the scene administrators and doers who understood the social value of advanced mathematics and were able to bring public resources to the recognition, support, and utilization of mathematics. They included Abraham Flexner, Oswald Veblen, T. C. Fry, Warren Weaver, Richard Courant, and Mina Rees. Finally there appeared popular interpreters of advanced mathematics such as Warren Weaver again and, later, the incomparable Martin Gardner. One would have to go back centuries to find a comparable list of prophets.

But perhaps the most important factor in the renaissance of mathematics in American scientific society was the idea of a mathematical model. It seems to have caught on with scientists and engineers in all fields. The public is now familiar with such models as the weather prediction model, as seen on TV. It is difficult to pinpoint just where the idea originated. One source seems to have been a seminal paper by G. A. Bliss in 1933 [39]. The computer was an essential tool to put the idea into practice, especially the many models that involved partial differential equations. The public could comprehend the idea of the mathematical model and understand the predictions it generated, with little or no technical understanding of the mathematics involved.

Postwar idealism and excellence, 1945–1954. When the war ended in 1945 the spirit of the country called for a better world to be achieved through competitive excellence. The government had plans ready to send the returning veterans to college, or graduate school, under subsidy of the GI Bill. The returning veterans, superimposed on the normal class of high school graduates, overflowed the universities. That created competitive conditions for admission and graduation. In this atmosphere of tough idealism the difficulty of mathematics was acceptable; mathematics was important. And there were not enough mathematicians to meet the demand. The public saw science and mathematics as keys to a better world. Only the people of the humanities felt neglected.

The idealism of the time was also expressed by an act of Congress which established the National Science Foundation (NSF) in 1950. Before it got into full operation, about 1952, the Office of Naval Research (ONR) conducted pilot operations in support of basic research in science and mathematics [40]. Other military research organizations joined in support of basic research.

Within the mathematical community a curriculum reform movement sprang up spontaneously, without any grants or commissions. Textbook writers came forth with new humanistic, or cultural, versions of freshman mathematics, and professors adopted them eagerly. There were books by F. L. Griffin, Morris Kline, Moses Richardson, Carl B. Allendoerfer and C. O. Oakley, and E. P. Northrop. The themes of these books included the cultural heritage of mathematics, the real numbers as a complete ordered field, logic à la Venn diagram and truth table, history, or unification of the technical components into a philosophical whole. They all made lower technical demands than the traditional texts; and this was their ultimate undoing. These were supposed to be terminal courses, but students in them kept showing up later in calculus, physics, or engineering where they were technically deficient. Yet in the postwar world mathematicians felt the need of their richer intellectual content.

Normally the universities have an excess of capacity in graduate studies and research because universities see this activity as conferring precious prestige. Normally they compete for graduate–research prestige with inadequate resources in qualified faculty and students, and always with inadequate funds. But the postwar era of excellence was a rare time when it was possible to establish new graduate–research departments and facilities without acting at the expense of one's rivals. However, it turned out that, if you were going to establish a new graduate department or institute, the time to do it was before 1954. After that it was too late; the old constraints began to return, though not all at once.

Around 1954 three little-noted events signaled the end of the era of excellence. First, the GI Bill veterans were phasing out about then. But now their numerous children, the war babies, could be seen and counted swarming through the lower grades. Clearly there would have to be a big expansion of college education to accommodate them, beginning in about 1963. That expansion would call for quantity rather than quality. Finally there was the 1954 Supreme Court decision (Brown *vs.* Topeka Board of Education) that outlawed segregation in the schools and guaranteed the right of public education without restriction by race.

This decision helped to justify a gradual shift from competitive excellence to concern for the underprivileged and underperformers in society. Mathematicians certainly shared this concern but it created special difficulties for mathematics. "Excellence" was still a good word and it took some years for somebody to invent the appropriate counter word, "elitism," to justify and mask the shift in national educational policy.

Policy and practice in the support of basic research in the early days of the NSF. The idea of a government foundation for the support of basic research was very new as the NSF got under way in 1952. There were no direct precedents to serve as guides for either policy or practice. The operations of ONR

came closest. And there were the long-established private foundations, such as the Rockefeller Foundation. The physicists and engineers had some experience in writing contract proposals for war research. But core mathematicians had none. Mathematics proposals, and therefore mathematics grants, were lagging behind when the first program director in mathematics was belatedly appointed [41]. His first task was to get more proposals for mathematics research that could qualify for support under the still undetermined working criteria. Getting the right reviewers was easier because *Mathematical Reviews* told who was doing research in each field and who was reviewing that work.

Although the National Science Act had directed the NSF to support research in basic science and mathematics, a number of crucial policy issues surfaced almost immediately. Those issues are still with us in 1988. They have only grown more complex, more relative, and more unresolvable. A history of the early NSF must recreate the innocence and the enthusiasm of those days, as well as the extreme caution imposed by the superlative first director, Alan Waterman. We were not aware in 1954 that, at that very time, the era of excellence and idealism, in which the NSF was established, was yielding to a new one emphasizing expansion and more expansion.

From the start much of the difficulty in both policy and practice came from the word, "basic." For "basic" is a relative term, not permitting a workable general definition. Without such a definition the discriminations for day-to-day operations in support of basic research are difficult, especially in mathematics.

More trouble came from the mathematician's need to buy time from teaching duties for his research. No assistant, or laboratory, can do his research for him. Scientists and engineers did not understand this. Their custom is not to pay the principal investigator's salary. Support comes in the form of laboratories, instruments, materials, and assistants. When exceptions were made for the mathematicians, the grants escalated to levels that brought out the cry: "greedy mathematicians!" And the NSF terminated payments of any portion of academic year salaries for principal investigators.

But more fundamental difficulties involved the principles governing the support of basic research in this new government foundation. Fortunately for mathematics, Chester I. Barnard, first chairman of the National Science Board, brought with him from his former position as president of the Rockefeller Foundation a passive policy of supporting proposals submitted by scientists. They were chosen for support only on merit, determined by peer review in the field. It was intended to forestall any NSF control of research through its money-granting power. Barnard's policy, which was almost religiously followed in the early days of the NSF, also helped to avoid questions about what was "basic research." It was good for core mathematics.

This passive proposal-support policy had its critics, including Abraham Flexner, himself a former head of the Rockefeller Foundation [29]. Flexner strongly advocated a mission-oriented policy such as the long-standing Rockefeller program to eliminate tropical diseases. And his philosophy had also led to the establishment of the Institute for Advanced Study, the choice of mathematics as its first subject, and the selection of eminent mathematicians for its first faculty. But such successes depended on having intelligent, wise, objective and idealistic minds to direct the mission, Flexner's and Veblen's, in the case of IAS, and then letting them direct it. With improper direction, or with political direction, mission-oriented research can go very far wrong. It is a policy better suited to a private foundation.

The argument was made moot by a decision of Congress to make a massive appropriation for an upcoming International Geophysical Year. And the NSF, which would administer it, wanted it because it would mean a big increase in the NSF's small budget. It dwarfed the program for the support of individual projects, even though that item was increased. In subsequent years a succession of such huge projects, some in space, some in national defense, were adjoined to the NSF budget. In mathematics this implied that money available for specific applications increased out of proportion to core mathematics support.

Before we leave this subject it should be remembered that there are other noninvasive policies, besides Barnard's proposal-support policy, that have been used to support basic research in mathematics. These have included the Sloan Foundation's support of promising young mathematicians, the NSF fellowship programs, and the support of AMS programs such as its research conferences and its publications. But the support of individual research proposals retains its crucial importance.

Educational expansion, 1954–1962. There was no sudden shift in policy in 1954. Just when the humanities might have been able to regain their lost domination of undergraduate education, the Russian satellite, Sputnik, launched in 1957, shocked the nation into a new national effort to compete with the Russians in space. Moreover the new National Science Foundation was just coming into full operation in 1952. But the main thrust of national effort had to be directed to expand colleges for the war babies. New state colleges were established, new multiple branches of state universities, and many new two-year community colleges. To teach the war babies would require that the production of new college teachers be tripled over the next ten years. That meant that we would have to dig three times as deep as normally into the relatively thin age class born between about 1927 and 1937 for graduate students. They did not have GI Bill financing. Since the humanities were expanding at about the same rate, science and mathematics could not, and in fact did not, get the extra graduate student manpower by luring students from the humanities. They had to come from the bottom of the barrel.

The NSF now extended its departmental grants program to the major state university departments which had existing graduate programs. Mathematics was now even more the area of greatest shortage of both faculty and graduate students. The grants that subsidized the expansion relieved the faculty for research, essential in a graduate program in mathematics. The teaching loads of the graduate students, actually teaching assistants, increased. So did class sizes; mathematicians preferred large lecture sections to small classes as a way of handling the requisite student load. Faculty research was promoted by these conditions; but graduate student research was not.

Mathematicians get back into school and college education. An unanticipated aftermath of the war was that PEB lost control of secondary education, at least partially, for the first time since 1900. American women had found jobs better paid than teaching during the war and now had other interests. In any case there was no postwar rush of women back to teachers' colleges. PEB could not supply enough trained teachers for the upcoming school expansion.

PEB had been ready with postwar plans which reasserted the old estimates: 20 percent college bound, 20 percent vocational education, and 60 percent in a curriculum now called "Education for life adjustment" [42]. It was a gross miscalculation. Both PEB's previous educational policies and the postwar plan were ill fitted for the national drive toward excellence through science.

Leaders in NSF felt that something had to be done to help the mathematics and science teaching corps. In 1952 the NSF funded a new venture, a Summer Institute for Mathematics Teachers at Boulder. It was led by Burton W. Jones, and E. Artin was the distinguished visiting mathematician. The next year there were two math institutes. Then the program spread to the other sciences. It became one of the most popular and successful of NSF programs, much liked by Congress [43]. Then the NSF expanded the idea to academic year institutes. These were associated with colleges of arts and sciences, with only apprentice teaching conducted by the school of Education. State legislatures suspended the requirement of a degree in Education for certification of teachers. A new degree was established in the graduate schools of arts and sciences, the Master of Arts in Teaching, MAT. These moves put mathematics and science departments, teamed with the NSF, full scale into the training of teachers.

Mathematicians were also leaders in curriculum studies to make better use of students' time and taxpayers' money than the now-decadent old curriculum and discipline did. While the main problem was clearly in the lower schools, the college curriculum was more accessible, and the college teachers did not have to be taught mathematics. That seemed the place to start, and the objective from the beginning was to eliminate as much remedial mathematics as possible so that college mathematics could start with calculus and/or discrete mathematics and probability.

CUP–CUPM. So in 1953, the MAA Committee on the Undergraduate Program (CUP) was organized, funded by a modest Ford Foundation grant. Its major published work was two sample textbooks for the first-year calculus and discrete mathematics, entitled *Universal Mathematics*, Parts I and II [44–45]. A Dartmouth group produced for CUP a version which was the precursor of the textbook by Kemeny–Snell–Thompson, *Finite Mathematics*, a pioneer for modern textbooks in this field [46]. In 1958 CUP was terminated, reorganized into special panels as CUPM, and funded by a substantial NSF grant as one of the several curriculum commissions in science that it supported [47–48]. The CUPM panels explored mathematics courses in support of the physical sciences and engineering, biological sciences, teacher training, computing, business applications, and pregraduate education for professional mathematicians. There was also a general curriculum in mathematics to support these special objectives [49]. One product of CUPM was an undergraduate textbook of contemporary engineering applications of mathematics [50]. The gathering of the material for this was a joint enterprise of CUPM and the Commission on Engineering Education; it could hardly have been done by private efforts.

CUP–CUPM never intruded much into the American public consciousness, and it was not to be expected that college mathematicians would accept the authority of any curriculum recommendations. Nevertheless, its influence was considerable, mainly from the concerted national effort towards curriculum reform that it generated.

Perhaps the best test of the success of CUP–CUPM is in the progress of early calculus and elimination of remedial mathematics in colleges. A study by the old CUP around 1953 showed that capable European students first study (not necessarily rigorous) calculus at about age 16. In the United States at that time few, even capable, students encountered it before age 19. By the mid-sixties great progress was being made to establish calculus, and/or finite mathematics, as the first college course, that is, at age 18. Then, around 1967, a breakdown began in high school preparation which ultimately forced the reintroduction of much remedial, precalculus mathematics in college. But even after that the amount of freshman calculus taught remained far above the 1953 level. And high school, or prep school, calculus is still (in 1988) on the increase, that is, calculus at age 17. A reasonable guess is that Americans who are going to study calculus at all now do so on the average at least a year earlier than in 1953. And many study it two years earlier. On the other hand, remedial mathematics in college is far from being eliminated. A large part of it is unproductive, a waste of students' time, of mathematicians' time, and of public funds for higher education.

SMSG. The School Mathematics Study Group (SMSG) was the official school mathematics reform commission formed by the mathematics division of NRC, and Mina Rees' old Policy Committee, in 1958. E. G. Begle was

asked to serve as its director. From the start it had a substantial grant as one of the curriculum study commissions supported by the NSF. Its board of directors represented the leadership of the Society and other mathematics organizations. For independence it was deliberately set up in a leading private university, first Yale and then Stanford.

There were several others of these curriculum groups in school mathematics supported by the NSF, including one headed by the somewhat flamboyant Max Beberman at the University of Illinois. Some of these proclaimed "the new math," but SMSG never claimed such a designation. In fact SMSG stayed within the confines of the traditional curriculum, grade by grade. Its operating membership included leading teachers from the public and private secondary schools as well as professors from the mathematics departments of teachers' colleges. With its resources SMSG was able to generate a great deal of national participation in the form of speaking, writing, conferences, and experimental teaching. Its membership included consultants from the other sciences and engineering. Moreover, the NSF conducted semiannual coordination meetings in which the commissions in all subjects met together and reported on their activities. No such massive and well-coordinated national effort has ever been mounted, before or since. The principal published output of SMSG consisted of sample text materials which it wrote for each of the grades. Its histories include a report in 1968 by Begle entitled "SMSG: The First Decade" [51–54]. It was the last decade; SMSG ended in about 1970, along with most of these educational reform commissions. We will return to that part of its history later.

Curricula for younger children. Two other commissions of the time were important. John Mayor, working at AAAS, conducted a study of mathematics in the primary and middle school grades, where many "math blocks" and deficiences are established. He addressed the important issue of the certification requirements in mathematics for *primary* school teachers [55].

Paul Rosenbloom at Columbia directed a commission studying the mathematical learning of quite young children, including preschoolers [56]. Many working on reform in mathematics education believed that those who later appeared as "disadvantaged" acquired their difficulties at preschool age. Begle, in particular, believed that remediation at age 18 was unproductive and wasteful of effort and money. He thought that this country had the resources and could not afford not to make a massive effort to educate disadvantaged mothers and their babies [57]. These judgments are reinforced by the widespread success of the Head Start programs for young disadvantaged children.

Research Potential and Training in the Mathematical Sciences. In 1957 that was the title of the joint AMS–MAA–NRC "Albert Survey" [58], which studied the conditions, favorable and unfavorable, for research and teaching in mathematics. Although NSF was making grants for research in 1957, the Albert Survey did not make more money for research its principal issue. In

fact one of its surprising findings was that heavy teaching loads, up to 15 hours per week and more, do not inhibit research production as much as expected. To be sure, the performance of young mathematicians in universities with light teaching loads was better. But those universities were usually able to select the best mathematicians for initial appointment. Perhaps the heavily loaded ones were better motivated!

A subcommittee of the Albert Survey considered the "small college effect," which is the observation that small colleges produce a disporportionate number of future research mathematicians. It took off from a classical NRC report by M. H. Trytten, entitled "Baccalaureate origins of the Science Doctorates" [59]. G. A. Hedlund, himself a product of such a college, wrote in the report: "There is one noticeable pattern which runs through the small colleges which have been highly productive of mathematicians. It is the concern for students as individuals and the opportunities open [to them] for growth. These can take many forms, but it seems clear that their effect is great" [60].

A 1988 retrospective, now that the small college has virtually disappeared, might call for an examination of college economics. In the College Blue Book, giving academic data on these colleges, one would find that the student/faculty ratio was almost uniformly 10/1 before World War II. Typically a student took five courses; so a teacher had a total student load averaging 50. A teacher taught these students in four small classes. The student got more individual attention than is possible today; and the teacher still had time to keep up a modest research program, without a grant, if he or she was so disposed. Many did, as the Albert Survey found. Tenure was usually informal, by custom rather than by formal action, and these colleges did not practice up-or-out at the tenure threshold. Even in the financially poorer colleges, where the student loads were higher, the student got much more teaching service than is considered possible today. These conditions undoubtedly had something to do with the productivity of the small colleges. But this kind of small college was fast disappearing when the Trytten report came out.

In the leading universities, with substantial endowment and grants income, the student/faculty ratio would be below 5/1, approaching 1/1. Only these institutions practiced up-or-out at the tenure threshold to control costs and maintain quality. Those dropped found ready jobs in secondary universities.

In the state universities with little endowment or grants income, where the students paid little or no tuition, the student/faculty ratio ranged from 12/1 to 15/1 and teaching loads of 15 hours per week were not uncommon. The same conditions obtained in the less well-endowed, tuition-sensitive, private universities. The Albert Survey showed that a mathematician could do some research without a grant under these conditions. And the students got reasonably good teaching service. These conditions had not eroded much under postwar conditions, where teachers were available, up to about 1954, because the GI Bill paid liberal, real tuition. The trouble in mathematics was that

the teachers were not available after the local supply of adjunct teachers was exhausted. Therefore the mathematics student/faculty ratio, and class size, tended to be higher than the university norm, and higher than those of English, which had a similar flood of freshmen. Unfortunately such ratios tend to become fixtures in the mathematics budget. Moreover things grew rapidly worse after 1954.

Community Colleges. A new type of college arose during the expansion. States usually set up the new two-year community colleges outside the system of state universities, with local boards of their own; yet they were teaching two years of college work that was guaranteed credit on transfer to the university. Initially they seemed to be more an extension of high school, and in PEB's domain, since many of their teachers were former high school teachers, often upgraded by the NSF academic year institutes. Later they got more Ph.D.s in their faculties and seemed to move towards college status. Mathematics was heavily involved in their identity crisis. They can teach remedial mathematics better than anyone else. But later many of the community colleges tried strenuously to escape the remedial teaching function. That left the problem of remedial mathematics unsolved. Also it would turn out that they often teach calculus better than the state universities. At first they rejected vocational education because of its low prestige but later found that they have a natural mission to train skilled technicians for electronics, health care, banking, accounting, computer programming, etc. [91].

CUPM tried to set up a program on the community college mathematics teacher, but was flatly rebuffed. In 1988 the proper relationship of the universities to the community colleges with respect to mathematical education is still unresolved. The community colleges, where one year of precalculus mathematics appears to be normal, not remedial, seem to have a clearer idea about it than have the state universities.

Manpower shortages 1962–1970. The Kennedy administration's ambitions in space brought to a head a conviction that had been growing among leaders of American business and industry. It was that our future prosperity in the new high-tech world depended on highly trained scientists and technologists, and that an acute shortage of such manpower was developing. Something had to be done about it, or we were headed for national decline [61]. This had been the feeling for some time in the mathematics departments of the universities. The Albert Survey had referred to "the critical shortage of mathematicians and teachers." The issue reached the government as a report of the President's Science Advisory Committee, the "Gilliland Report," 1962 [62].

The report called for a massive expansion of Ph.D. production in the fields of engineering, mathematics, and physical science. It was especially critical of mathematics, which was deemed to be underproducing both in undergraduate majors and in Ph.D.s. It set a hoped-for 1970 goal for mathematics Ph.D.s

of seven times the 1960 output of 303 Ph.D.s, or, more realistically, at least four times. It called for large government appropriations and for financial help from the private sector to meet these goals. In a preface to the report, President Kennedy appealed to the patriotic spirit of young Americans to continue their education to an advanced stage. He said in part: "... It is the students themselves who hold the key to this nation's strength. It is my earnest hope that each college student will consider how valuable additional study will be in enhancing his abilities and potential contribution to the nation, and in bringing him greater satisfaction and rewards."

The immediate response of the mathematicians was expressed in a 1963 conference conducted by the Conference Board of Mathematical Sciences (CBMS) [63]. We felt important but skeptical of those big numbers. We wanted to do what our President asked, but nobody knew where we were to find these Ph.D.s. The amazing thing is that the *maximum* goal of the President's Commission, 2200 by 1970, was almost reached by 1999 mathematical science Ph.D.s in 1970–71 [64]. It had been 303 in 1960.

One of the responses of NSF was to establish a program of support to "Centers of Excellence," that is, to universities that had not previously granted significant numbers of Ph.D.s. This further aggravated the real shortage of mathematicians who were qualified for doctoral research and advanced teaching. Salaries of mathematicians went sky high by previous standards. The numbers of service-free NSF fellowships was not increased in proportion to the new goals. Those graduate students were needed as teaching assistants. Often the teaching loads of these teaching assistants were quite high. Also classes became large lecture sections, previously considered not good practice in freshman courses. Finally, even the youngest of these American graduate students in 1963 were coming from the low birth rate years before 1941. That meant that we were reaching well below levels of ability previously considered minimal for graduate study, we were overloading them with teaching, and we were teaching them with professors who had not previously directed doctoral research.

Actually they turned out better than we had any right to expect. But there had been no planning for the type of training they would receive. Mathematicians were mathematicians. Of necessity these expansion Ph.D.s were educated in core mathematics, and not much else. That limited their usefulness and employability to the academic sphere. Even there they were too narrowly educated to be the best teachers.

The President's Commission in 1962 set off a great deal of monitoring of scientific manpower production besides that of the U.S. Office of Educational Statistics [66]. In mathematics the 1967 Young survey (John Jewett, Executive) did the most thorough reporting of quantity and quality in mathematical education and manpower production [67].

The mathematicians of this time have a good record for objectivity when the national mood screamed "shortage." The famous 1971 Allan Cartter report, just before the crash, praised the Young–Jewett survey for its "sober and objective analysis" [68]. Somewhat earlier CUPM had considered a possible special project on an alternative to the Ph.D. as qualification for college teaching. But it rejected the idea on the advice of its special committee's hard-headed report by Lowell Paige. And in 1969 W. L. Duren, speaking to the conference of academic deans on their problem of shortage of mathematics faculty, urged them to wait and not to start any more Ph.D. programs in mathematics. Mathematicians would soon be available. They did not believe him.

COSRIMS. In 1968 the Committee on the Support of Research in the Mathematical Sciences (COSRIMS), with Lipman Bers as chairman, issued its report [65]. It had been set up as a ten-year followup to the Albert Survey [58]. But it was more specific in its objectives, as the name indicates. Its survey of education and manpower was delegated to the Young committee [67]. And its own subcommittee on undergraduate mathematics issued a separate report. The COSRIMS report was the subject of sharp debates within the mathematical community. It is difficult to gauge its influence in seeking more public support for research in mathematics since it came out just when the mathematics boom was about to collapse. And much of the NSF money had to go for an expansion of the postdoctoral research program for unemployed Ph.D.s. One of COSRIMS's small, but lasting, contributions was the phrase, "core mathematics."

Alternative graduate education in mathematics. As the shortage of mathematicians built up in the fifties, both for education and for industry, grave doubts arose about the existence of enough young people to satisfy the demand for Ph.D.s. For they had to have both the mathematical talent and the motivation to meet the research requirement of the Ph.D. This feeling was strong years before President Kennedy's 1963 call for a huge expansion of Ph.D. production. An obvious solution was a doctoral level graduate degree for college teachers with some substitute for the research dissertation. Such degrees had been given before, both in Germany and the United States, at least as early as 1888, often called "Doctor of Science." For the proposed doctoral level degree for college teachers Don Wallace proposed the name, "Doctor of Arts." There was much emotional feeling both for and against the idea. The issue came to a head in an official conference at Yale on October 21, 1962, for which Edwin Moise had prepared the presentation [69]. Richard Brauer spoke for the opposition, saying that "it would water down the Ph.D." The opposition won and the D.A. was dead, for a time at least.

The 1961 rejection of the teachers' D.A. degree did not settle the issue. The D.A. that was rejected was too narrowly a vocational degree for college

teachers, described mainly negatively as a relaxation of the research requirement. It did not spell out the educational qualifications for either a fully educated college teacher or a teaching assistant. (These questions were addressed in CUPM reports of 1967 and 1969 [49, p. 102, p. 113].) And the D.A. proposal did not admit that the education needed by a college teacher is very similar to the advanced education needed by mathematical practitioners in science, industry, and government. Combining these objectives could produce a graduate education to equip the student for college teaching, or practice as an applied mathematician, or later specialization in some research field of the mathematical sciences.

In 1969, with the prospect of an oversupply of Ph.D.s looming, suddenly the question was: Are there too many Ph.D.s? R. D. Anderson, in the first of many counts he made of supply and demand, concluded: Not yet, but.... W. L. Duren said in effect: Not if we maintain high standards and broaden their education to equip them for a wide range of jobs and usefulness in society [71]. Lipman Bers, in the COSRIMS report, held the traditional research degree line. Others proposed many variations on these themes.

And, if we may look ahead in this history, even after the Ph.D. glut hit in 1971, the discussion of reforming the content of national graduate education has continued with a variety of motivations [98]. A panel of CUPM, chaired by Alan Tucker, developed the idea some years later as an undergraduate major program called: A General Mathematical Sciences Program [70]. But that idea never got very far because such curricula for generalists are too difficult at the undergraduate level. It might have had more chance as a professional Master's degree competing with the M.B.A. in business, or the Master's in computer science. The surplus of the Ph.D.s continues to block the acceptance of such reforms.

It is well to look at how the problem has been resolved in other academic fields. In physics, chemistry, and biology, all sciences with distinct theoretical, experimental, and developmental emphases, one Ph.D. suffices, although it represents for individuals quite different kinds of study and research, or development. In medicine the practitioner's degree is a doctorate, the M.D., and the academic research degree, the Ph.D., is an extension of it. A similar situation holds in law. These make research education very long and costly to finance. But in all these fields, as well as in engineering, the number of graduate practitioner's degrees conferred is always several times as many as for academic research and graduate professors. In mathematics the insistence that the only nominally three-year graduate degree education shall be the one represented by the traditional, academic research Ph.D. severely restricts not only the number of mathematicians but also the usefulness of mathematics in American society.

Suddenly from shortage to glut. The end of the mathematics boom in this country came with great suddenness in 1971. It had come a little earlier in

physics. The trigger was the Allan Cartter report [68], based on demographic data which showed that the war baby birth rate had already topped out, that the previous linear projections for the future demand for Ph.D.s as science and mathematics teachers were excessive, and that in 1981 college enrollments would begin to decline. The argument applied in the humanities just as well, since they had experienced the same growth boom that mathematics had, except that they had not been artificially inflated to three times their stable equilibrium share of college enrollments. Even so, why so sudden? It was still ten years to 1981.

But it was sudden. The 1999 Ph.D.s in 1971 found few jobs of any kind, certainly not the university positions they had anticipated. Many found no academic jobs at all. In desperation some took to driving taxicabs. Trying to help them, the AMS formed the Committee on Employment and Educational Policy. CEEP studied the evidence and could only say that it would not go away; it was not a short-term recession that would be followed by a renewal of growth. The NSF closed down the Center of Excellence grants and all the curriculum study commissions. Apparently it did not ever decrease its budget in support of basic research. But more of it had to go as postdoctorals for unemployed Ph.D.s, leaving assistant professors unsupported. Not only did the universities stop the employment of new Ph.D.s; they terminated assistant professors at the tenure threshold where they could legally do so. The terminated ones could not find other academic positions, as such rejects had done in the past. They were 35 years old, many had families, and now they had to start over in a new job where their mathematical education might be inappropriate. University administrations also tried to accelerate retirements of senior professors. If they could terminate one of these, it would save a larger salary. The new buzzword became "tenured in," something to be avoided at all costs.

But why was the collapse so sudden and so severe? The Cartter report had predicted that maximum enrollment would be reached in ten years. The main reason was the difference in the definition of "demand" between President Kennedy's commission and the Cartter report. In 1962 "demand" was conceived as a national need, not associated with visible salaried jobs. In 1971 Cartter defined it narrowly as predicted jobs in college teaching. And the prediction was based on an assumed student/faculty ratio of 25/1 instead of the prewar standards of from 10/1 up to 15/1. It also left out industry and government employment because that could not be quantified. The difference was enormous and the result cataclysmic.

Another factor was the vanishing of marginal income. The faculty expansion had been financed largely by the marginal tuition revenue from enrollments, increasing each year. When the Cartter report proved that enrollments would level off, this marginal income did not level off; it vanished suddenly.

There were other contributing reasons. When the war babies got to college they did not share the parental generation's ideals of excellence through competition, in which they were now the competitors. They turned activist against the Vietnam War, the draft, nuclear arms and energy, and against the science and technology that implemented war. That included mathematics. In the universities they found friends in the teachers of the humanities, who now perceived a chance to regain some of their lost importance. The informal coalition was joined by PEB, which wanted and got the curtailment of science education grants by the NSF in favor of educational grant programs out of the U.S. Office of Education. These events released the pent-up resentment of millions who could not compete, or did not choose to compete, in this postwar technological world.

5. RETRENCHMENT, 1970–1988?

Retrenchment in the university administrations. Before we look at retrenchment in mathematics departments we should look at what university administrations did to counteract the effects of the sudden termination of money for expansion. For one almost immediate result was that mathematics departments were forced back into dependence on their university budgets more than when the department and members had grants from an outside agency. The research grants were still there but the educational grants for expansion of graduate production in mathematics were not.

As we have seen, university presidents' first concern was to cut back on salary commitments to tenured faculty. There was a lot of false reasoning about the "tenuring in" issue. Mathematicians were able to bring some rationality to the problem of how to get the unbalanced faculty flow back into a steady state. J. P. LaSalle [72] and John Kemeny [73] produced Markov chain models of faculty flow that showed what management strategies would get the faculty back into the desirable steady state and with the least hardship.

After that, in a rational world, one would expect the presidents to close down costly and unproductive programs of graduate work and research in science and mathematics, quietly, with no public announcement. Generally speaking that is not what happened. Instead, presidents of universities with new marginal graduate programs clung to them. Having lost their Center of Excellence grants, they had to finance their graduate programs out of general budgets for undergraduate teaching, plus research grants that their professors could bring in. They increased pressure to publish and get a grant. Publication was made a necessary condition for tenure in universities that had never required it before. The teachers to be cut, or have their salaries and rank frozen, were the older professionals who had taken positions as undergraduate teachers before the new graduate program was installed. Their teaching

was replaced by "graduate teaching assistants" who were usually just low-paid, locally available adjunct teachers. Moreover, these conditions obtained even in colleges that still did not have graduate programs. Many of them are still pressing to establish one.

Many state universities now moved toward open admission policies in order to maximize enrollments and therefore the subsidy paid by the state in lieu of tuition. They could make a "profit" by admitting as many first-year students as possible and teaching them cheaply in large sections with low-paid adjunct teachers or graduate teaching assistants. They could do this at cost-per-student-semester-hour below the state subsidy. And the profit could be used to subsidize expensive graduate programs. This policy implied a great deal of remedial instruction as the quality of performance of high school graduates deteriorated. Much of the burden of this remedial teaching fell on the mathematics and English departments. The profit from this remedial instruction is, of course, a fictitious one created by the practice of unit cost accounting. In fact, such remedial instruction adds to the total cost which has to be paid, and diverts educational funds to unproductive efforts, actually leaving less for advanced teaching and research. The state councils of higher education are beginning to find this out. For example, a recent study in Virginia makes public the $30 million that remedial instruction is costing each year in the state, even with a lenient definition of "remedial." So mathematics departments should be warned that this way of earning money probably will not last [74].

Privately endowed universities and some well-financed state universities pursued the opposite strategy of maintaining admission standards. To them, remedial instruction is obviously pure cost. This proved to be a winning strategy in the competition between universities for good students.

Retrenchment in the mathematics departments. Math departments had more than their share of young people of professorial rank when the PhD. glut struck. Those who did not have tenure were cut mercilessly, even many promising and productive ones, because the department already had more than was deemed its share of tenured positions held over from the days of the math boom. This applied even in the top departments. In all but the top ranking departments graduate students in core mathematics became very scarce. To keep their research seminars active the faculty talked to each other about their research, or brought in visiting mathematicians. Academic year relief from teaching duties was no longer provided by NSF grants. But free time for research is important to a mathematician, so mathematicians generally chose to do their teaching in a fewer number of large lecture sections, often with quiz sections attached that teaching assistants handled. The system is legitimate, if well managed, but is subject to abuse. In some state university math departments the student/faculty ratio rose far above the 25/1

that was the basis of the Cartter report calculations. This is poor teaching service, especially if the professor is devoting much time to research.

Mathematics departments feel that this system was thrust upon them by the events following the 1971 crash. The relief they feel they need is more grant money to support core mathematics research. But even if the money were available, there is a question whether this solution would improve the overall service of mathematics in American society. It is one of the tough problems remaining in the wake of the 1971 crash.

PEB resumes control. The recognition of the cresting of the war baby population took the pressure off the demand for high school teachers. It did not produce a glut like the Ph.D. glut because the women had not returned to the teachers' colleges. In fact, enrollments in these colleges would continue to decline, forcing many of them to convert to liberal arts colleges with only a department of education. But PEB was able to supply the reduced demand. PEB got $887 million of National Defense Education Act funds, formerly assigned to the NSF, transferred to the U.S. Office of Education control [75]. The NSF curriculum study commissions, including SMSG and CUPM, were terminated. The activity of mathematics departments in curriculum and teacher training came to an abrupt halt. PEB's new curriculum plan to replace "Education for Life Adjustment" still called for 20 percent college bound, 20 percent vocational, and 60 percent in a terminal liberal curriculum that was to be more like a free elective system. A number of new subjects with contemporary flavor were to be added to the old ones. To restrain the chaos of a free elective system in high school, a new position, guidance counselor, was established in each high school. The guidance counselors received some additional training and higher pay than the teachers. That was where the NDEA funds went.

If anyone proposes more required courses, PEB's answer is to challenge the speaker for daring to dictate what free American students should study. Yet this is exactly what the guidance counselors often do. This happened again recently when Secretary of Education, William Bennett, proposed a curriculum for James Madison High School in which every student would take a course of study that included the old academic subjects covering the 15 Carnegie units for admission to college [76]. It included three years of mathematics. His argument was that this was the best education our civilization has been able to offer. The immediate challenge from the president of the American Federation of Teachers reasserted the old PEB estimate that such a curriculum is for "15 to 25 percent" [77]. The actual experience as of 1985 is that by ages 20–24 only 16 percent have less than 12 years of school, 46 percent finished high school but did not attend college, 41 percent had some college, and 23 percent had four or more years of college [78].

These figures imply that thousands of students are entering college without the standard college preparatory course, but with the terminal PEB education

instead. Thousands more are entering college, nominally from the college preparatory program, but with little mathematics or science, if they have expressed a preference for the humanities to their guidance counselors. On the other hand, those students that choose careers in science or engineering may come to college with extra credits in mathematics, including high school calculus. There are many more of these students from private preparatory schools. The total is still relatively small but they make a large fraction of those entering the leading schools of science and engineering. And this system is sending mathematical illiterates to top liberal arts schools and even to the science and engineering programs of the many state universities. The situation is similar to the disorder which existed in college education after the introduction of Eliot's free elective system around 1900 and had to be corrected by stronger curricular direction. This is the basis for Secretary Bennett's call for a return to a secondary curriculum in which about two-thirds is a required common core of traditionally proven value, including three units of mathematics.

Did SMSG fail? A minor skirmish in PEB's campaign to recapture control of primary and secondary education involved the termination and discrediting of SMSG and the new math. The older mathematics teachers, trained in education schools, were unable to adjust to the new text materials and protested in mounting furor. Other critics, including Morris Kline among the mathematicians, furnished support to the claim that the SMSG curriculum had failed. Parents who could not help their children with their homework joined the clamor. This made PEB's takeover easier, but the main factors that ended SMSG were PEB's national political moves that terminated all of the NSF curriculum study and teachers' institute programs, together with the demographic changes that enabled it to resume supplying the teachers from its own schools of education, temporarily at least. But members of the mathematical community have apparently taken the criticisms of SMSG as the reason for its downfall, and still feel that its errors must be corrected to restore the good name of mathematics.

Actually SMSG did not fail. It did not have a life long enough for a definitive test of its first-edition text materials, whose obvious flaws included an excessive enthusiasm for logical language. With minor, necessary, exceptions, SMSG stayed in the traditional curriculum. Its advisers were leading university mathematicians who ensured that the mathematical content of its program was sound. Its great success lay in the massive national reform effort that it mounted, in the lively corps of teachers it and the institutes produced, and the enthusiasm for mathematics that they generated in their students. After the PEB takeover the old math teachers made life very uncomfortable for the SMSG–NSF trained teachers. Many of them left to get better-paying jobs in business where their computer training was an asset. Others moved

up to the faculties of community colleges. But others stayed, and many of them are teaching high school calculus now.

If there are any lingering doubts about SMSG as a cause of the deterioration in arithmetic skills among high school students, one need only look at what happened after PEB restored the old math. Then things really got worse! To be fair, the real cause of the breakdown in school mathematics performance can hardly be attributed to the math curriculum, old or new. PEB has more serious problems with failure of discipline, drugs, crime, gang warfare, teenage pregnancy, and general chaos.

PEB loses control again. The schools of education are still dissolving, or converting to liberal arts colleges. PEB no longer has its monopoly on teacher training. The teachers are taking control of the National Education Association and turning it into a union [92].

The American public is aroused by evidence that American workers, educated in our schools, are inferior to their foreign competition, are causing the country to lose its competitive strength, and to lose jobs. Quality education will be an issue in the next presidential election. The only reason that this has not come to a head earlier is that the private schools educate the children of business, political, and industrial leaders. At the moment the focus is on the teachers. Businessmen think we can afford to pay good teachers more if we just identify the good teachers. Actually, the teachers are the main strength of the system. Something more fundamental is wrong.

The history of American education indicates that the decision of 1893 has become a mistake and needs to be changed. Mathematics is especially involved. Recall that in 1893 the decision was made, setting up the American high school under local school board control with no quality control either by the universities or the federal government. That worked well enough when only a small fraction of public high school graduates went to college; and most preparation for college was done by private academies, whose quality could be monitored by the College Entrance Examination system. But now that about half of public high school graduates go on to college, and state universities are expected to accept them, that arrangement is working badly. A large fraction of public funds for higher education is going into unproductive remedial teaching. And these remedial students are being poorly served by universities using them to support unproductive graduate programs with their fictitious tuition profits. It might be tolerable if the education of the noncollege-bound students was successful, but their dropout rate and the illiteracy rate among those students denies that.

The time has come when we must change the system to provide a much stronger direction of studies, and much stronger quality control of college preparation, than PEB and the politically vulnerable local school boards can

manage by themselves. As recounted above, we mathematicians tried unsuc-
cessfully to get that stronger direction of study, through graduation require-
ments, discussed in the report of the J. W. A. Young committee of 1923, and
in several lesser efforts. We tried, with only temporary and limited success,
to do it with the weak instrument of curriculum recommendations by CUP–
CUPM and by SMSG. Now the time appears to have come for us to go back
into the national political arena, urging the establishment of a general studies
curriculum, such as the one that Secretary Bennett proposed for Madison
High. That includes sufficient mathematics so that high school graduates can
begin college mathematics with calculus, or discrete mathematics with prob-
ability. It will serve the nation and the students better, whether they go on
to college or not. We ourselves should speak for mathematics, not leaving it
to the engineers and physicists to speak for us.

American mathematicians and political issues. The course of events since
World War II has forced American mathematicians to modify their long-
standing, somewhat apolitical, attitudes. After all, we say, the business of the
American Mathematical Society is mathematics: the promotion of research,
publication, and teaching in mathematics. Our majority attitude had been:
If mathematicians want to play politics, that is their right; but let them do it
in the open public domain, not in the mathematical organizations. Still, as a
group, we mathematicians have a characteristic political makeup that differs
from that of the average educated American. On the whole mathematicians
are more ignorant and unsophisticated about politics and its arts. The tra-
ditional issues of academic freedom have not touched mathematicians very
much. Not since Galileo has mathematics been regarded as a subversive doc-
trine, although there were times when the new math appeared to reach that
status.

We tend to be conservative in the sense that we treasure the right of the
individual to do his thing. Many of us are military-oriented because of the
long involvement of mathematics in military science. But we also tend to
be internationalists since mathematics is an international culture, indepen-
dent of language and politics. This characteristic has sometimes got us into
trouble, or at least made us suspect as security risks.

The trouble started during the Eisenhower Administration (1951–1959),
when some mathematicians were brought before the McCarthy or Velde
(House Un-American Activities) committees, charged with being Commu-
nist "fellow travelers" at least. Others were fired from their tenured, civilian,
positions in state universities because they refused to sign a loyalty oath as
a condition for employment. Thus the issues of academic freedom came to
mathematics.

AMS and MAA formed some joint committees to investigate and defend
those fired. In at least some cases the real charge turned out to be homo-
sexuality, an unmentionable in those days. The investigating committee was

supposed to understand why, when the facts were revealed confidentially, the charge had to be stated as communist affiliation. But these ad hoc committees could do nothing effective to reverse any such dismissals. So AMS–MAA formed the Committee to Prevent the Loss to Mathematics of those Dismissed for Political Reasons. This committee heard the victims' stories and tried to find jobs for those dismissed. It did not attempt legal defenses. Very few learned societies attempted to give financial support to litigation in these cases.

During this same period evidence came out about the persecution of particular mathematicians or scientists in the Soviet Union and in other totalitarian countries. Resolutions in their defense were presented in AMS business meetings. Members hotly debated these resolutions, not on the merits, but on the propriety of AMS involvement. Several such resolutions passed [93].

Then came the 1971 crash, followed by sharp reductions in tenured faculty commitments. This could be accomplished in several ways: by denying tenure to assistant professors, by forced early retirement, by breaking tenure on grounds of hardship, or by freezing rank and salary for selected individuals. The affected mathematicians often naively believed that academic freedom gave them a constitutional right like the First Amendment freedom of the press. Actually the courts had never given academic freedom the status of constitutional law. But, in the era of litigation that followed, it turned out that the AAUP Statement of Principles of Academic Freedom and Tenure [79] did have a form of legal status. The enforcement of federal laws, such as those governing breach of contract, fair employment practices, or race or sex discrimination in the workplace, all depend on an accepted definition of fair employment practices in each industry. For the academic "industry" the only generally accepted such statement was the AAUP Statement of Principles. Both AMS and MAA, as well as most other learned societies, colleges, and universities had subscribed to it. Thus the AAUP rules of tenure became an essential instrument in the enforcement of federal law in these cases, even if they themselves did not have the status of law. This launched a second stage in the use of the rules of tenure: the protection of job security in the courts, independent of traditional issues of academic freedom.

Once the rash of litigation after 1970 had cleared, and college administrations had legally established the commitment to the Statement of Principles, a third stage in the use of the tenure principles evolved. Colleges, as well as graduate universities, denied tenure, or advancement, to those who did not publish research. Thus the rules of tenure became an instrument of job insecurity, as we have seen above. (We will return below to the special political issues involving women as mathematicians.)

The emergence of NSF support of research, with its issues of distribution of finite funds, also introduced hot political issues into mathematical society. The political issues may involve funds for mathematics as opposed to physics,

funds for core mathematics research as opposed to applied research, basic research as opposed to mission-oriented research, "regional development" as opposed to highest quality, or education as long-range research support as opposed to a particular current research project. NSF itself steers assiduously clear of such issues, but they are unavoidable in the advisory panels. The American mathematician's old-time innocence of politics appears to be lost forever. And now we must overcome our timidity and get into the battle over the national high school curriculum and its mathematical content.

How to organize a national effort in mathematical education or support of research. The social history of mathematics in the years 1945–1970 records efforts of unprecedented scale to improve our conditions and our national service. There are some lessons to be learned from that history about the alternative ways to proceed in such ventures. A brief listing of them with their advantages and disadvantages may be useful.

In the dual, private–public system of American education, the private sector offers some advantages of prestige and freedom from leveling political forces. So when SMSG was set up, it was located in a leading private university, first Yale and then Stanford. The disadvantage was that no private university has the financial or manpower resources to conduct such a big project. All of that had to be brought in and financed by a foundation grant. When the grant was terminated the project was dead.

CUP–CUPM chose to locate in the association as a standing committee. This has the disadvantage that professional associations are not unbiased; they represent the interests of their membership. Consequently foundations do not like to make grants to projects so located. CUPM was able to overcome this, and get an NSF grant, only by assuming a dual status as an NSF commission. But it had the advantage of the mathematical manpower resources of MAA, the audience at meetings, and the publication facilities. While the NSF grant was terminated, CUPM has continued to live as a standing committee of MAA, though its activities are limited by financial constraints.

Another possible location is in the mathematics division of the National Research Council. This has some unique advantages since NAS–NRC is the official interface between the institution of science–technology and the goverment. It enjoys the prestige of NAS and unquestioned objectivity in matters of science. Since the NSF was established, NRC has traditionally conducted the screens of fellowship applicants. Moreover it has permanent government financing of its own, relatively small, operations. It is the ideal place to initiate a public project that seeks NSF financing. For example, SMSG was initiated in NRC. Its disadvantage as a place to operate is that it is small, its appointed membership visits only at brief meetings, and it is more oriented to research than to education. It loses some authority in the field of education since it represents only science education. At the present time

the mathematics division of NRC houses both our research grant advisory function and the Mathematical Sciences Education Board.

By analogy, and by structure, the proper interface with the government for mathematical education should be the American Council on Education. But ACE has developed in a different way. So far it has not served as a base for efforts in education involving mathematics.

Both the physicists and the chemists have established Washington institutes that represent the combined membership of their various professional associations. Such an institute can perform the function of a lobby, or represent the profession other ways, especially in dealing with the government. This representation can be frankly in the professional interest of the scientists, without the tradition of objectivity that NRC has. We mathematicians tried to establish our own such institute called The Conference Board of Mathematical Sciences (CBMS). Baley Price was its founder. It served well during its existence, but not all of the mathematical science organizations were willing to participate. AMS and MAA, combined, find themselves too small to afford it. CBMS was able to exist for a time on grants, but when these gave out CBMS folded. We now have only Kenneth M. Hoffman as our part-time representative in government matters. He has proved to be the best communicator we have had in this status.

The NSF itself has mounted educational efforts in mathematics, as well as other sciences. These have included the summer and academic year institutes for teachers. And the fellowship programs have to be included. But, unlike the research grants, the NSF has not invited individual proposals in education; it has pursued its own missions in education, advised as always by NRC, or ordered by Congress. Also, when it had funds for such projects, it has supported large projects proposed by mathematical organizations. These included several major CBMS projects.

It may come as a surprise to mathematicians, but the NSF is not the only foundation making grants. In particular there are private foundations; and projects funded by them enjoy some of the prestige and freedom from political influences that private universities have. It is true that, when the NSF was formed to support basic science, there was an informal understanding that the private foundations would support the humanities. But mathematics is in large part a humanistic culture. In fact the first grant of $75,000 to support the old CUP in 1953 came from the humanities program of the Ford Foundation. The Foundation would have granted much more but, in the quaint innocence of 1953, the Executive Committee of MAA thought that such a large grant might unduly influence mathematics. More important is the Sloan Foundation fellowship program, selecting promising young mathematicians for support without specific project competition. And in the specific areas of applied mathematics, the military agencies and others

too numerous to mention, have been, and continue to be, active in research support.

Private corporations have also helped. For example, around 1950, General Electric printed and distributed millions of copies of a pamphlet by their W. E. Boring called: "Why Study Math?" More recently American Telephone's Henry Pollak has spent a lot of company time as a sort of roving ambassador to mathematics. These voices offer a different, and often more influential message than that of academic mathematicians. IBM has also been a friend of mathematics, as have the other computer companies. One gets the impression that corporations would have done more to advance study and research in mathematics if we could have found the right ways for them to do it.

The College Entrance Examination Board [7] is a nonprofit association with many colleges and universities, both private and public, as members. Its data provide the best objective assessment of the quality of secondary school performance. This capability will be even more important in the future if we are to reform secondary education to eliminate wasteful, demoralizing, remedial mathematics teaching in college. Beyond its testing program, CEEB set up a commission on Mathematics (1955–1958) with A. W. Tucker as chairman, to determine what they should be testing for in the mathematics achievement test. Its report had lasting influence on the content of mathematics in good secondary schools.

We have excellent sources of statistical data on education in this country. The Division of Educational Statistics of the U.S. Department of Education has a fine tradition in this field. Its C. B. Lindquist was especially active in helping the mathematicians. The Research Division of the National Education Association (NEA) has provided the data on teacher supply and demand. And the Department of Labor regularly studies the supply of trained labor and the jobs for them.

The unofficial Professional Education Bureaucracy (PEB), the administrative divisions of the U.S. Department of Education, and the central power structure of NEA have been like enemy territory to the mathematicians. SMSG and the NSF institutes' programs were able to enjoy friendly cooperation with NEA's math subsidiary, the National Council of Teachers of Mathematics (NCTM). But this relationship deteriorated after PEB resumed full control of public school education and forced the termination of SMSG, as well as the NSF institutes, in 1970. In the large grants program for research in education that the U.S. Office of Education set up after that, there were projects funded in mathematical education, but apparently not many to members of a university mathematics department. The recent political activism of the teachers, which got cabinet status for Education and turned NEA into a labor union, is a good omen for the future. The teachers can be counted on to be stronger for educational quality than their bosses in PEB

were. If the teachers really control NEA, we mathematicians can work with them, as we did with NEA's subsidiary, NCTM, during the days of SMSG.

Recent realignment of our relations to American society. Although CBMS continues to exist as a council of presidents of the mathematical sciences organizations, Kenneth M. Hoffman is now our designated spokesman to the federal government. Our public representation, both for research policy and education, is now concentrated in NRC [80]. These arrangements are designed to develop our part of the recommendations of the 1984 David Report viewing mathematics as a "Critical Resource for the Future" [83]. It is too soon to write any history, but to this historian it looks good.

6. Some Unfinished Stories

Women in graduate mathematics. It is obvious that mathematics does not come in male and female genders; but women as mathematicians have always been a special category in the American mathematical community. Their status has changed back and forth in response to conditions in American society. Their complete social history has not been recorded; and a part of it is still conjectural. The following might be a reasonable, if oversimplified, scenario.

In 1888 there were not enough graduate students for all the new American graduate programs in mathematics. So women were most welcome, then as they are today. On the other hand the demand for the graduates in paying and satisfying jobs was much less urgent. They could teach in the elementary and, soon to be developed, high schools or in the female colleges.

Then, after the 1920 women's suffrage amendment to the Constitution was passed, there was a great surge of professionalism among American women in which many became advanced mathematicians. But by the end of the decade the difficulty of getting university positions in competition with the men became discouraging. The Great Depression clinched it. Meanwhile PEB had established a virtually closed shop control of teaching jobs in the schools. These provided more security than university positions and were better suited to place-bound women. So most women mathematicians settled for education degrees and school teaching. That was the state of affairs until World War II when women found better paying and more satisfactory jobs in war work. And after the war they were busy raising families.

The production of the war-baby generation was about complete in 1963 when President Kennedy called for a massive, subsidized expansion in advanced engineering, mathematics, and physical sciences,. Mathematically talented women eagerly responded to his call. These women, returning to graduate mathematics, were a major reason for the unexpectedly high quality of these expansion Ph.D.s. But they had come back, only to be caught in the retrenchment after 1970. After 1970 many mathematical women switched to

computers, to M.B.A. degrees, and to professional positions in business. Others found jobs in community colleges. They still did not return to schools of education. A surprising number, however, stayed in graduate mathematics. In university faculty positions the ratio of women to men is still increasing. University regulations now more freely permit husband and wife to be members of the same faculty.

All this did not happen without organized struggle. In this phase the name of Mary Gray has been prominent.

In this changing picture there is one invariant principle. It may be stated in terms of the, still valid, concepts of 1963, viewing mathematical brain power as a precious national resource. If half of the mathematically talented minds are female, then, for the national welfare, half of the fully educated and fully utilized mathematical minds should also be female. That has never become a reality in this country.

Foreign graduate students. Besides the women, the other unanticipated source of mathematical talent that made the crops of expansion Ph.D.s after 1963 better than we had any right to expect came from abroad. Their numbers have been increasing year by year, relative to native-born Americans, until in 1987 more than half of American Ph.D. degrees in mathematics were awarded to foreign students [81]. This happened in spite of the exclusion of foreigners from most fellowships, and in spite of the language difficulties of supporting them as teaching assistants. They do not come for the humanities. They have their own humanities. They want engineering, science, mathematics, and computer science. There are differing views as to whether this foreign invasion is good or bad.

Mathematics departments want them. Outside the very top departments, there are not enough good graduate students to keep the graduate programs functioning in a healthy manner. These students are not only selected for ability from a world pool of mathematical talent (excluding only the Soviet Bloc countries), they also tend to be better trained in certain areas such as hard analysis and mechanics. This may make them better than Americans in applied mathematics. There are many more in engineering than mathematics, and their registration in graduate courses helps to maintain a full program of advanced mathematics. Finally, we want these students because of our feeling that we and they belong to the world community of mathematics.

Managers of medical research laboratories, industrial, and government laboratories want these graduates and want them to stay in this country. We could hardly operate medical research or hospitals, or our research and development establishments, without them.

On the other hand a congressman, a taxpayer, or an industrialist may regard this as a shocking giveaway of costly American science and technology at taxpayer expense. For even if they pay full tuition, these students pay only

a fraction of the high cost of their graduate education. Then they may go home to set up high-tech industry in competition with the American companies and their workers, whose know-how and financial resources created the technology they studied.

Still another view of this phenomenon has been expressed by J. J. Servan-Schreiber, former French cabinet member and writer on international economics [82]. His view is that America must remain the world's graduate university for the sake of both U.S. and world economic, technological, and intellectual development.

Clearly American mathematics has a stake in this socioeconomic issue, however it may ultimately be resolved.

Another way to find support for basic research in mathematics. There are now many able, well-trained, young mathematicians whose research proposals cannot be supported by the NSF for lack of funds. It is obvious to mathematicians that the relief needed is more money to support proposals in basic research. What could be wrong with an appeal for more grant money to help us complete our personal mathematical work, to relieve us of teaching, and to increase our annual income?

In fact there are cogent public policy arguments against providing more grant money to relieve mathematicians of teaching while the student/faculty ratio remains as high as it is now. Since a grant does not pay academic year salaries, it does not directly fund more research. Indirectly, its payment to the university can provide a teaching assistant to relieve the researcher of some academic year teaching. But the student/faculty ratio is already too high; and the students are getting only thinly diluted teaching service from professional mathematicians. This situation could only be made worse by using more teaching assistants. The grant cannot provide what is really needed—another tenured faculty position. In short, as long as the professional mathematics faculty remains too small, more research can only be bought at the cost of poorer teaching service.

History suggests another way to support basic research in mathematics as a corollary of a better educational policy. It is an accompaniment, not an alternative, to grants in support of research proposals. This is simply to reduce the student/faculty ratio back to the traditional prewar levels ranging from a standard 10/1 up to 15/1 in teaching colleges and universities.

As the Albert survey of 1957 showed, mathematicians could do creditable research while teaching 12 to 15 hours a week [58]. But the total number of students they taught was low, based on prewar student/faculty ratios. During the postwar period of explosive growth in enrollments, faculty shortages, and price inflation, academic economics could not sustain the old quality ratios. Postwar staffing called for student/faculty ratios such as Allan Cartter's norm of 25/1, and up from there. But now that enrollment is stabilized, and

faculty manpower is in abundant supply, state universities should return to traditional quality standards in college education. Other universities and colleges will follow them.

Graduate professors cannot teach 12 hours. "Graduate professor" does not mean one who teaches a "graduate course", but one whose principal occupations are doing and leading research and writing, directing doctoral dissertations, editing and refereeing research institutes. For such professors a teaching load of three to six hours is appropriate, as it was before World War II when the basic college teacher's norm was 12. A graduate professor's position is commonly called a "chair," whether its reduced teaching is accounted for by endowment, grants, or other budgetary provision.

A first-class graduate department needs several such chairs; and its student/faculty ratio is below 10/1. It is therefore very expensive. They cannot be proliferated as political patronage in every town, like community colleges. A large state university may be able to combine a good graduate and undergraduate department, but not without sacrifice of quality in one, or both, components when the overall student/faculty ratio is 25/1 or higher. Much harm has been done to U.S. higher education, both in quality and runaway cost escalation, by the continuing of too many pseudograduate departments after the onset of the 1971 Ph.D. glut and the subsequent proper discontinuation of the federal Center of Excellence grants. These pseudograduate departments are propelled by local pride and financed by abuses of low-level service and remedial teaching.

The main burden of undergraduate teaching should be carried by well-qualified professors, instructors, or legitimate graduate teaching assistants, whose primary duty is teaching. For them the full-time basis of 12 hours is appropriate. And, for fairness, their expected performance in research and publication should be less that for chaired professors, but still greater than zero. The rules for these discriminations have to be worked out locally, within departmental or university government.

The cost to states to lower student/faculty ratios will be high, and it will take time to implement. But there is no reason why postwar society cannot sustain prewar standards of education for its youth. Mathematicians would do well to join in public pressure for it. In doing so we would find that we have many friends, including English professors and their humanities colleagues, as well as many in the general public, especially women. By contrast, in an appeal for more research grant money for mathematics, we are isolated, and few in number.

Although research support would be only a secondary objective in going to a larger faculty, let us consider the ways in which it could help mathematical research. First, it would make more jobs for fully trained mathematicians. More mathematicians could do more mathematics. And we have many good

mathematicians, women especially, who merit university positions. Admittedly the prewar faculty policy was less efficient as stimulator of research than NSF-style support of reviewed proposals. But it was a policy that left the mathematician free to work on what he chose. Moreover, contemporary pressure to publish as a condition for tenure would improve its efficiency. And that pressure would be more fair than it now is, as applied to an overloaded faculty, where grant-getters have all the advantage. These are reasons, based on history, why mathematicians should now give a higher priority to enlarging the faculty than to increasing research support for the existing overloaded faculty.

The appreciation and status of mathematics. We have worried for a hundred years that we and our subject are not understood and appreciated by the American public. Only recently Murray Gerstenhaber has expresssed our plight eloquently and philosophically [89]. He is only the latest to do so. Over the years the diagnosed causes have been manifold: not enough money for basic research, poor public relations in the media, not enough applied mathematics, and so on. But the lesson of history points to one basic cause. As E. B. Wilson put it in 1913, "We do not make good" [for the average American out there].

Consider the many millions of Americans whose collective viewpoints comprise our public image. Their view of mathematics is primarily determined by their experiences in their last, most advanced, course. For the vast majority of them this was a terminal course, some version of high school or college algebra, taught as a service course to prepare them for something else [90]. The most turned off of them must be the ones who got to state universities and were herded into large sections taught cheaply to earn budget credit for the support of the department's research program. Consider, even, the smaller, elite fraction whose terminal mathematics was a huge, monolithic, three-semester calculus course loaded down with too many good things. It was also primarily a service course. Will one of these former students understand or appreciate mathematics?

It is not that we have not tried. As we have seen in this history, we have tried, and are continuing to try, to solve this problem. Some departments, and some fine teachers, have succeeded. But the numbers of their fortunate students are small in comparison with the multitudes that were batch processed. We have not solved the problem in the aggregate; and that is our public image problem. We can blame PEB and John Dewey for a lot of it. We can blame our university administrations for some of it. But we ourselves are still to blame for much of it. We could have learned something from John Dewey about the motivation of young students and not depended as much on compulsion as we still do. But until we have educated a whole generation of Americans for whom mathematics makes good in their own experience, we will not have solved our "image problem."

The public understanding and appreciation of mathematics is related to our status in society, but it is not the same. Status is more subjective. Is it the same as prestige? The history has less to say about status. Surely it is not to be expected that improved teaching service will solve our status problem, though it might help. Certainly our status in American society took a great leap from limbo in the years after World War II. Indeed, as Abraham Flexner found out earlier, old mathematics had never lost it in high intellectual circles. Lately, after 1971, we perceive ourselves to have lost some of our postwar gains in status.

If we insist on regarding our status as a problem to be solved, then history tells us that we have an identity problem which should be solved first. We are hanging between the humanities on one hand, and science and technology on the other. If we hitch our wagon to the technology star, then we are forever a service subsidiary. But if we claim our rightful place in the ancient humanities, we are put down as materialists, and excluded from the club by such Johnny-come-latelies as the English and history professors and the snobbish literati outside of academia, bent on enhancing their own precarious status. History might sigh and say that there is not much we can do about it.

AMS and mathematical education throughout life. We come last to the newest, the most exciting, and explosive developments of the century in mathematical education, and the service of mathematics to American society. These are in the field of education continuing throughout life. They are beyond, or outside, the framework of formal education with its curricula, courses, lectures, credits, degrees, and its faculty.

The idea of education continuing throughout life is very old. But educators in the academic institutions have treated it more with lip service than with imagination. They have relegated it to an "extension division" of low prestige and often questionable quality. In mathematics its main product has often been evening courses in college algebra. An occasional new idea would show up, such as the sixties idea of canning lectures on film or videotape. But these electronic extensions of the devices and personalities of formal education never got very far.

Over the centuries the time-honored instrument of education throughout life has been the book, and then the library to make books accessible. During this century in higher mathematical education the most important institution of continuing education has been the American Mathematical Society, joined later by its sister societies in mathematics, applied mathematics, statistics, logic and computer science. The AMS programs that promote research, publish research, and disseminate mathematical knowledge through meetings, have enabled mathematicians to grow and develop after their formal education was complete. Under NSF support we have developed summer conferences, or institutes, in which mathematicians from around the world can cooperate in research in a chosen field. Finally, we must remember that

in 1930 the Institute for Advanced Study became a great institution of continuing education in mathematics, outside the framework of formal education.

These developments have special significance in graduate education for applied mathematics. The nature of the subject is such that an applied mathematician's formal education could never be either extensive enough, or intensive enough, to meet his future needs. Now, as never before, his formal education need not be definitive. Later, in practice, he can continue his education, keep up with progress in his field, and grow as his problems demand. This was always the ideal but was never so possible to realize as it is now. Some of the new developments that make this possible include not only the research conferences, but short courses such as those that were offered by the Mathematics Research Center at Madison, Wisconsin, and now Cornell, and desktop publishing with the aid of Donald Knuth's $T_{E}X$ software.

But the most explosive developments in continuing mathematical education are coming at a more popular level, in vocational mathematics. Mathematicians have been slow to get into this field. Medicine, with its changing technologies and strong financing, has long engaged in continuing education to keep its practitioners up to date. Agricultural extension services have made new technologies available to farmers. Now, belatedly, the uses of the computer have drawn mathematicians into it. Every meeting of AMS has associated short courses or demonstrations of mathematical applications of the computer. Also, with the computer itself as teacher, and with the aid of commercial software, many thousands of Americans are using mathematics as never before. Users of "spread sheets" are doing matrix algebra at high speed, even if they do not know what a matrix is, or that they are doing matrix algebra. Where this all leads, what its implications are for mathematics in American society, no one knows. In any case, from the highest mathematical level to the lowest, informal education, continuing throughout life, is where things are happening. And that is where AMS teaches.

References and Notes

[1] Flexner, Abraham, *Universities, American, English, German.* 1930.

[2] Hofstadter, Richard and C. DeWitt Hardy, *The Development and Scope of Higher Education in the United States,* 1952.

[3] Kandel, I. L. *History of Secondary Education,* 1930. Also his article "Secondary Education" in Encyclopedia Brittanica (1966).

[4] Kandel, I. L. *The New Era in Education,* 1955.

[5] Conant, James B. *Education and Liberty,* 1953.

[6] Conant, James B. *The American High School Today,* 1959.

[7] Valentine, James A. *The College Board and the School Curriculum,* College Entrance Examination Board, New York, 1987.

[8] For example, in the years 1945 to 1965 life in a mathematics department was dominated by the returning veterans with their G.I. Bill, then the phasing out of the veterans by the advent of the National Science Foundation, then Sputnik, the coming of computers, and the expansion of education to accommodate the war babies, all events belonging to the history of higher education.

[9] The written histories of American mathematics before 1888 tend to be social, rather than intellectual, histories, because the main activities of this period were teaching and scholarship. Research was just beginning. On the university level we have Cajori's 1890 survey **[10]**. And, mainly on the school level, there is D. E. Smith and J. Ginsburg **[11]**. There is also an excellent recent article by Judith Grabiner **[13]** which has the flavor of social history.

[10] Cajori, Florian, *History and Teaching of Mathematics in the United States.* U.S. Government Printing Office, 1890.

[11] Smith, David Eugene and J. Ginsburg, *A History of Mathematics in America before 1900*, Chicago, 1934.

[12] Tarwater, J. Dalton (Editor), *Bicentennial Tribute to American Mathematics, 1776–1976*, Math. Assoc. of America, 1976.

[13] Grabiner, Judith V. "Mathematics in America, The First Hundred Years," in Tarwater **[12]**.

[14] Flexner, Abraham, *Daniel Coit Gilman*, New York, 1946, 29.

[15] James, Henry, *Charles W. Eliot (1834–1926)*, 2 vols. 1930.

[16] Kandel **[3]** 472 ff.

[17] Kandel **[3]** 468 ff.

[18] Valentine **[7]** 3 ff.

[19] Conant **[6]**

[20] Kandel **[3]** 486–491.

[21] Mach, Ernst, *Science of Mechanics*, 9th edition, 1933, trans. by T. J. McCormack, Open Court Pub. Co. 1942. In the preface Ludwig Mach quotes E. Mach as saying: "I do not consider the Newtonian principles as a completed and perfect thing, yet in my old age I can accept the theory of relativity just as little as I can accept the existence of atoms and other such dogma."

[22] Moore, E. H. "On the Foundations of Mathematics," *Bull. Amer. Math. Soc.* **9** (1902-3), 402. Also *Science*, **17**, March 1903, 401–416.

[23] Wilson, E. B. "Let us have calculus early," *Bull. Amer. Math. Soc.* **20** (1913), 30–36.

[24] Birkhoff, Garrett, "Some Leaders in American Mathematics." In Tarwater **[12]**.

[25] Kandel **[3]**, 449. Public high school enrollments grew from 0.9 million in 1910, to 1.9 million in 1920, to 4.2 million in 1928. And that was before the World War I babies arrived.

[26] I can report first hand on the bad effects of that postwar expansion. In 1918 I entered a new expansion high school, where a rapid succession of inept college students "taught" me algebra. In 1920 my parents moved to New Orleans where I transferred to an old, but also expanding, high school. The teacher in advanced algebra promptly sent me back to first-year algebra where Mr. Theriot, an old professional with a Master's degree from Tulane, rehabilitated me, at no extra compensation I am sure.

Without his generous help I could not have become a professional mathematician. Mr. Theriot was working with me after class one day when a group of the new teachers passed by. He muttered his disdain for them. To him they had no class and did not belong. Then he explained what had happened and why these unqualified teachers were there. The place was going to the dogs!

Later, in college, I became a tutor and teaching assistant, teaching remedial mathematics to graduates of similar expansion schools. My students were not fortunate enough to have been rescued in time by Mr. Theriot. And I did not succeed in saving many, if any, of them by remedial teaching in college.

[27] Young, J. W. (editor), *The Reorganization of Mathematics in Secondary Education*. A report of the National Committee on Mathematical Requirements, under the auspices of the Mathematical Association of America, MAA 1923.

[28] In my first encounter with PEB, I went with my Chairman to Baton Rouge in 1926 to meet the Louisiana state officers of education. Professor Buchanan urged that Louisiana adopt the Young Committee recommendations for a mathematics requirement for high school graduation. The officials treated us with bemused tolerance but deaf ears. They let us know that we had no influence whatever, while they had the political power. So why had we come all the way from New Orleans?

[29] Flexner, Abraham (with Esther S. Bailey) *Funds and Foundations*, 1952. Also recounted in *I Remember: An Autobiography*, rev. ed., 1960.

[30] The Institute for Advanced Study, *A Community of Scholars, Faculty and Members, 1930–1960*. Princeton, 1980.

[31] Montgomery, Deane, "Oswald Veblen," *Bull. Amer. Math. Soc.* **69** (1963), 26–36. Reprinted in [94], 118.

[32] Lax, Peter, "The Bomb, Sputnik, Computers, and European Mathematicians." In [12].

[33] Rees, Mina, "The Mathematical Sciences and World War II," *Amer. Math. Monthly* (1980), 607–621. Reprinted in [94], 275.

[34] Rosser, J. Barkley, "Mathematics and Mathematicians in World War II," *Notices Amer. Math. Soc.* (1982), 507. Reprinted in [94], 303.

[35] Brothers, Leroy A. *Operations Analysis in World War II*. Philadelphia, 1946. Also Charles W. McArthur, *Operations Analysis in the U.S. Army Air Forces in World War II*, to be published.

[36] Goldstine, H. H. *The Computer from Pascal to von Neumann*, Princeton, 1972.

[37] Feller, William, *Introduction to Probability and Statistics*, vol. 1, 2nd ed.,. New York 1957, 6, 31, 138. Feller gives an account of these ideas in the introduction.

[38] Weyl, Hermann, *Group Theory and Quantum Mechanics*, 1928, xxii.

[39] Bliss, G. A. "Mathematical Interpretations of Geometrical and Physical Phenomena," *Amer. Math. Monthly* (1933), p. 472.

[40] Rees, Mina, "Mathematics and the Government, the Postwar Years." In Tarwater [12].

[41] This account is based on recollections of my service as first program director for mathematics in the NSF. Bob Stoll had represented mathematics before me when it was included in physics. But in late 1952, at the urging of Marston Morse, then on the National Science Board, mathematics was given its own program. My secretary and I made up the whole section. What little I knew about the job I had learned from Mina Rees, but now she was unavailable for counsel because she was program

director in that rival agency, ONR! We were all extremely cautious not to overstep our congressional authority. I used to read the National Science Act daily. Director Alan Waterman's staff checked everything I did, or wrote. But the program directors in physics, chemistry, and engineering could not have been more friendly. And the mathematics community was terrific! In spite of the constraints, I think that Leon Cohen, who succeeded me, and I had more freedom to launch new initiatives than the program directors who came after. It was a great time to be in the NSF.

[42] National Education Association, *Education for all American Youth*, Report of the NEW educational policy committee, 1944. Also *Vitalizing Secondary Education*. U.S. Office of Education, Bull. No. 3, 1957.

[43] Krieghbaum, Hiller and Hugh Rawson, *An Investment in Knowledge*. NYU Press, 1969. (A history of the NSF institutes for teachers.)

[44] *Universal Mathematics, Part I, Functions and Limits*, CUP-MAA Lawrence, KS, 1954; *Part II. Elementary Mathematics of Sets*, Robert L. Davis, Ed., CUP-MAA, Charlottesville, VA, 1958.

[45] Duren, W. L. Jr. "CUPM, The History of an Idea," *Amer. Math. Monthly* **74** Part II, 1967, 23–37.

[46] Dartmouth College Writing Group, *Modern Mathematical Methods and Models*, 2 vols. MAA, 1958.

[47] Gehman, H. M. "The Washington Conference," *Amer. Math. Monthly* **65** (1958), 475.

[48] Davis, R. L. ed., *CUP Collected Reports and Transition Report*, MAA, 1957.

[49] Mastrocola, William E., ed., *A Compendium of CUPM Recommendations*, 2 vols. MAA, 1975.

[50] Noble, Ben, *Applications of Undergraduate Mathematics in Engineering*. MAA and Macmillan, New York, 1967.

[51] Begle, E. G. "SMSG, The First Decade," *The Mathematics Teacher*, March, 1968.

[52] Wooton, William, *The Making of a Curriculum*, Yale Univ., 1965. (A history of SMSG.)

[53] Jones, Phillip S., Ed., *A History of Mathematical Education in the United States and Canada*, 32nd Yearbook of NCTM, 1970, specifically the article by Alan R. Osborne and F. Joe Crosswhite, "Reform, Revolution and Reaction," 268–281.

[54] Jones, Phillip S. and Arthur F. Coxford, Jr., "Mathematics in the Evolving Schools." In [53] 11–92, especially Reaction, 81–83.

[55] Mayor, John. See the reports of the AAAS Committee on the Teaching of High School Science and Mathematics. Also see the reports of Mayor's Maryland Project.

[56] I cannot find the record of Paul Rosenbloom's work on mathematics for very small children, in an NSF commission at Columbia. It was based on the work of Jean Piaget, particularly his *The Child's Concept of Number*, New York, 1952. Other follow-up studies to Piaget's work are reported in [53] and in a later NCTM yearbook.

[57] Based on recollections of several conversations with Begle on what SMSG should do about remedial education, particularly in underprivileged sectors of society. Also see E. G. Begle, "The Role of Research in Improvement of Mathematical Education," Proceedings of the First International Congress on Mathematical Education, Dordrecht, Holland, 1969, 106.

[58] Albert, A. A. *A Survey of Research Potential and Teaching in the Mathematical Sciences*, 2 vols. Univ. of Chicago, 1957.

[59] Trytten, M H. *The Baccalaureate Origins of the Science Doctorates Awarded in the U.S. 1936–1950*, NAS–NRC, Pub. 382, Washington, D.C. 1955.

[60] Hedlund, G. A. *Report of a Conference on Undergraduate Mathematics* (Held at Hunter College), New Haven, 1957.

[61] Proceedings of the Sixth Thomas A. Edison Foundation Institute, *The Growing Shortage of Scientists and Engineers*, N.Y.U. 1955.

[62] Gilliland, Edwin R., Ed. "Meeting Manpower Needs in Science and Technology," Report Number One of the President's Science Advisory Committee: *Graduate Training in Engineering, Mathematics, and Physical Sciences*, The White House, Washington, D.C. 1962.

[63] Cohen, Leon W., ed., *Manpower Problems in the Training of Mathematicians*. Report of a CBMS Conference, Washington, D.C. 1963.

[64] Annual Publication, 1974 edition, National Center for Educational Statistics, *Projections of Educational Statistics to 1983–1984*. Table 25. U.S. Department of Health, Education, and Welfare.

[65] COSRIMS, *The Mathematical Sciences, A Report*, Committee on the Support of Research in the Mathematical Sciences, Lipman Bers, Chairman. Also *The Mathematical Sciences, Undergraduate Education*, National Academy of Science, Washington, D.C. 1968.

[66] Annual Publication, U.S. Office of Scientific Resources, *Doctoral Recipients in U.S. Universities*, Washington. Also Annual AMS Survey, *Notices Amer Math. Soc.*, Providence, and various NSF reports.

[67] Jewett John, ed. CBMS Survey of the Mathematical Sciences, Gail Young, Chairman.

    I. John Jewett and C. B. Lindquist, *Aspects of Undergraduate Training in the Mathematical Sciences*, Washington, D.C. 1967.
    II. John Jewett, L. J. Paige, H. O. Pollak, and G. S. Young, *Aspects of Graduate Training in the Mathematical Sciences*, 1969.
    III. J. P. Lasalle, C. R. Phelps, and D. E. Richmond, *Aspects of Professional Work in the Mathematical Sciences*, 1970.
    IV. John Jewett and C. R. Phelps, *Undergraduate Education in the Mathematical Sciences*, 1970–71. Washington, D.C. 1972.

[68] Cartter, Allan M. "Scientific Manpower for 1971–1985," *Science*, **172** (1971), 132–140.

[69] Moise, E. E. "The Proposed Doctor of Arts Degree," *Notices Amer. Math. Soc.* **8** (1961), 112. Conference at Yale on the proposed Doctor of Arts degree, Oct. 21–22, 1961. *Resolutions*. Apparently not published.

[70] Tucker, Alan, Chairman *Recommendations for a General Mathematical Sciences Program*, CUPM Report, MAA, Washington, 1981.

[71] Issues concerning the Ph.D. *Amer. Math. Monthly*, **77** (1970). R. P. Boas, "The COSRIMS reports," p. 623. R. D. Anderson, "Are there too many Ph.D.s?," p. 636. W. L. Duren, Jr. "Are there too many Ph.D.s?" p. 641. Alex Rosenberg, "Some further thoughts on the Ph.D. in mathematics," p. 646. Rosenberg refers to an earlier article, I. N. Herstein, "On the Ph.D. in mathematics," *Amer. Math. Monthly* **76** (1969).

[72] LaSalle, J. P. "Appointments, promotions, and tenure under steady state staffing," *Notices Amer. Math. Soc.*, **19** (1972), 69.

[73] Kemeny, John, "What every college president should know about mathematics," *Amer. Math. Monthly*, **80** (1973).

[74] McCartin, Anne Marie, and Margeret Miller, Report: Costs of remedial education in Virginia, VA. State Council on Higher Education, March, 1988.

[75] Johanningsmeier, E. V. *Secondary Education*, Academic American Encyclopedia, Grolier, 1985, describes how funds from President Eisenhower's 1957 National Defense Education Act were used "to train guidance counselors and administer tests that would identify those students who had the appropriate aptitudes for such studies" [i.e., for the science, mathematics and languages specified by NSDA]. I cannot find the original sources documenting the new slogan of "relevance" and the guidance counselor program, and once more rely on memory.

When the USOE grants program was set up with NSDA funds, largely replacing NSF education programs, I felt that mathematicians should cooperate. I served on various USOE committees and as a "field reader", i.e., referee, on many proposals. I often worked with Kenneth Brown of USOE, whom I liked and trusted. The end came for me in a USOE conference set up ostensibly to advise USOE how the NSDA funds should be allocated. I found that it was a sham; and that the decision had already been made to use the funds on the new guidance counselor program. It was the first I had heard of it. I found the program director in charge arrogant in his power, as no NSF program director would have dared to be. I stormed out of the meeting in protest. Later I got a soothing letter from the Commissioner of Education himself, but he did not ask what my grievance was; and UDOE never again asked me to serve.

[76] Bennett, William, "The Ideal Curriculum for James Madison High School," *New York Times*, Dec. 30, 1987.

[77] Shanker, Albert, Pres. Am. Federation of Teachers, "Bennett's James Madison High School," *New York Times*, Jan. 3, 1988, p. E9.

[78] Digest of Educational Statistics, 1987, USDE, Washington. Table 9, Years of Education Completed by Age, U.S. Bureau of Census, Sept. 1986. The figures quoted are based on the age class, 20–24, including delayed admissions to college. We can get some idea of the fraction of 18-year-olds who entered college directly by comparing Table 69, for the numbers of 17-year-olds graduating from high school in 1984-85, with the numbers of entering freshmen that fall. Assuming that 25 percent of community college freshmen are in degree-credit curricula, the numbers are: 27 percent dropout or delayed, 35 percent temporarily terminated high school graduates, and 38 percent entered degree credit programs in college. These figures will converge towards the ones given for the age class 20–24 as these students grow older. Actually freshmen admissions in some kind of college in the fall of 1985 were 85 percent of high school graduates the preceding spring. These figures do not support Shanker's and PEB's claim that the standard college preparatory curriculum is appropriate for only 15 to 25 percent of students.

[79] "1940 Statement of Principles of Academic Freedom and Tenure," published annually in the AAUP Bulletin, Washington, D.C.

[80] Hoffman, Kenneth M. "Washington Outlook," Periodic columns published in *Notices Amer. Math. Soc.* and MAA *Focus*, especially *Notices* **35** (1987), 898.

[81] Connors, E. A. "Annual AMS Survey," *Notices Amer. Math. Soc.* **35** (1988), 525–534.

[82] Servan-Schreiber, J. J. and Herbert Simon, "America Must Remain the World's University," *Washington Post*, Outlook, Nov. 15, 1987.

[83] David, Edward E. Jr. *Renewing U.S. Mathematics: Critical Resource for the Future*, NAS–NRC, Jan. 1984, "The David Report."

[84] Archibald, R. C. *A Semicentennial History of the American Mathematical Society, 1888–1938*, AMS Semicentennial Publications, vol. 1, 1938.

[85] Birkhoff, George D. "Fifty Years of American Mathematics," in AMS Semicentennial Publications, vol. 2, 1938.

[86] Chandrasekhar, S. "Einstein and General Relativity; Historical Perspectives," *Amer. J. Phys.* **47** (1979), 212.

[87] Dyson, Freeman, *Infinite in All Directions*, New York, 1988, p. 62.

[88] See Goldstine [**36**], 279–286.

[89] Gerstenhaber, Murray, "On the Status of the Mathematical Profession," *Notices Amer. Math. Soc.* **31** (1984), 469.

[90] Digest of Educational Statistics, USDE, Washington, (Annual) Also [**67**] vols. I and IV.

[91] Conference Report, "New Directions in Two-Year College Mathematics," *Notices Amer. Math. Soc.* **32** (1985), 176.

[92] "Virginia Praised for Revamping Teacher Degree," *Charlottesville Daily Progress*, August 19, 1988. As I write this the state board of education has tentatively approved a proposal to replace the education bachelor's degree by the liberal arts degree for the certification of all Virginia teachers.

[93] Pitcher, Everett, *A History of the Second Fifty Years of the American Mathematical Society, 1939–1988*, AMS, Providence, pp. 297–306.

[94] Duren, Peter, ed. with Richard A. Askey and Uta C. Merzbach, *A Century of Mathematics in America, Part I*, AMS, Providence, 1988.

[95] Reingold, Nathan, "Refugee Mathematicians in America, 1933–1941; Reception and Reaction," In [**94**], 175.

[96] Lefschetz, Solomon, "Reminiscences of a Mathematical Immigrant in the United States," In [**94**], 20.

[97] Bers, Lipman, "The European Mathematicians' Migration to America" In [**94**], 231.

[98] Nohel, John A. "Some Thoughts on the Role of Applications in the Development of Ph.D.s in Mathematics," *Notices Amer. Math. Soc.* **22** (1975), 380.