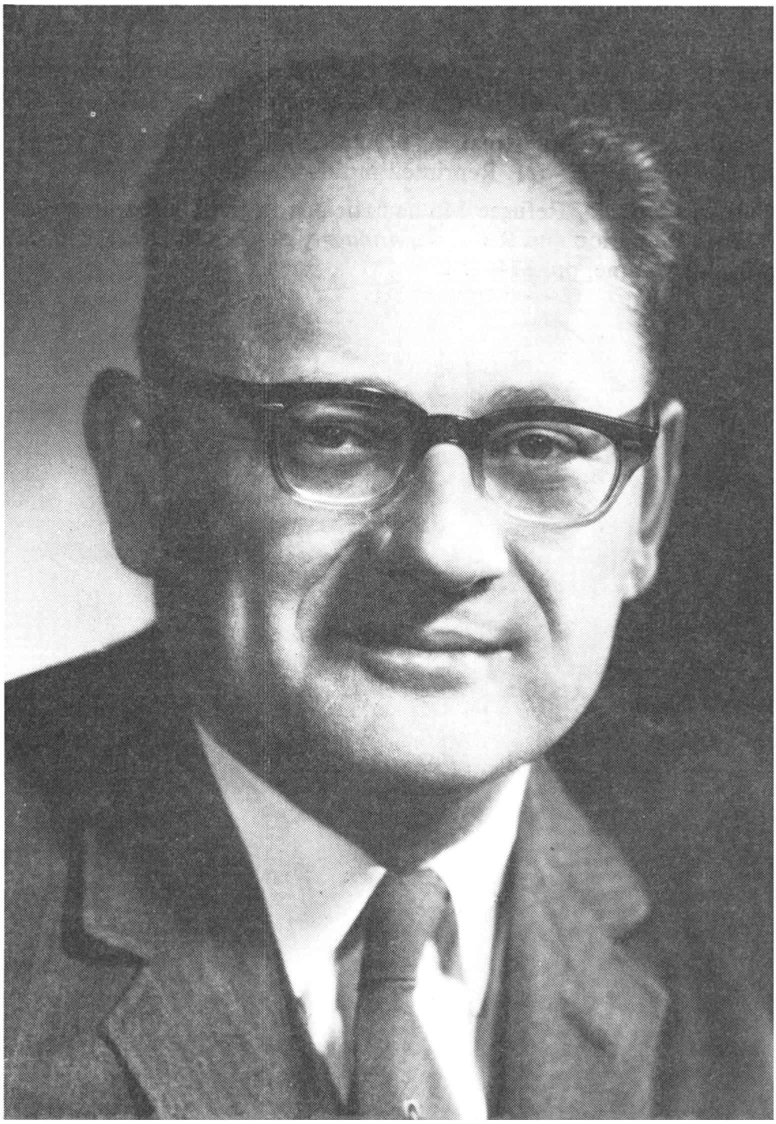

A. Adrian Albert

ABRAHAM ADRIAN ALBERT

November 9, 1905–June 6, 1972

BY IRVING KAPLANSKY

ABRAHAM ADRIAN ALBERT was an outstanding figure in the world of twentieth century algebra, and at the same time a statesman and leader in the American mathematical community. He was born in Chicago on November 9, 1905, the son of immigrant parents. His father, Elias Albert, had come to the United States from England and had established himself as a retail merchant. His mother, Fannie Fradkin Albert, had come from Russia. Adrian Albert was the second of three children, the others being a boy and a girl; in addition, he had a half-brother and a half-sister on his mother's side.

Albert attended elementary schools in Chicago from 1911 to 1914. From 1914 to 1916 the family lived in Iron Mountain, Michigan, where he continued his schooling. Back in Chicago, he attended Theodore Herzl Elementary School, graduating in 1919, and the John Marshall High School, graduating in 1922. In the fall of 1922 he entered the University of Chicago, the institution with which he was to be associated for virtually the rest of his life. He was awarded the Bachelor of Science, Master of Science, and Doctor of Philosophy in three successive years: 1926, 1927, and 1928.

On December 18, 1927, while completing his dissertation, he married Frieda Davis. Theirs was a happy marriage, and she was a stalwart help to him throughout his career. She remains active in the University of Chicago community and in the life of its Department of Mathematics. They had three children: Alan, Roy, and Nancy. Tragically, Roy died in 1958 at the early age of twenty-three. There are five grandchildren.

Leonard Eugene Dickson was at the time the dominant American mathematician in the fields of algebra and number theory. He had been on the Chicago faculty since almost its earliest days. He was a remarkably energetic and forceful man (as I can personally testify, having been a student in his number theory course years later). His influence on Albert was considerable and set the course for much of his subsequent research.

Dickson's important book, *Algebras and Their Arithmetics* (Chicago: Univ. of Chicago Press, 1923), had recently appeared in an expanded German translation (Zurich: Orell Füssli, 1927). The subject of algebras had advanced to the center of the stage. It continues to this day to play a vital role in many branches of mathematics and in other sciences as well.

An *algebra* is an abstract mathematical entity with elements and operations fulfilling the familiar laws of algebra, with one important qualification—the commutative law of multiplication is waived. (More carefully, I should have said that this is an associative algebra; non-associative algebras will play an important role later in this memoir.) Early in the twentieth century, fundamental results of J. H. M. Wedderburn had clarified the nature of algebras up to the classification of the ultimate building blocks, the *division algebras*. Advances were now needed on two fronts. One wanted theorems valid over any field (every algebra has an underlying field of coefficients—a number system of which the leading examples are the real numbers, the rational numbers, and the integers mod p). On the other front, one sought to classify division algebras over the field of rational numbers.

Albert at once became extraordinarily active on both battlefields. His first major publication was an improvement of the second half of his Ph.D. thesis; it appeared in 1929 under the title "A Determination of All Normal Division Algebras in Sixteen Units." The hallmarks of his mathematical personality were already visible. Here was a tough problem that had defeated his predecessors; he attacked it with tenacity till it yielded. One can imagine how delighted Dickson must have been. This work won Albert a prestigious postdoctoral National Research Council Fellowship, which he used in 1928 and 1929 at Princeton and Chicago.

I shall briefly explain the nature of Albert's accomplishment. The dimension of a division algebra over its center is necessarily a square, say n^2. The case $n = 2$ is easy. A good deal harder is the case $n = 3$, handled by Wedderburn. Now Albert cracked the still harder case, $n = 4$. One indication of the magnitude of the result is the fact that at this writing, nearly fifty years later, the next case ($n = 5$) remains mysterious.

In the hunt for rational division algebras, Albert had stiff competition. Three top German algebraists (Richard Brauer, Helmut Hasse, and Emmy Noether) were after the same big game. (Just a little later the advent of the Nazis brought two-thirds of this stellar team to the United States.) It was an unequal battle, and Albert was nosed out in a photo finish. In a joint paper with Hasse published in 1932 the full history of the matter was set out, and one can see how close Albert came to winning.

Let me return to 1928–1929, his first postdoctoral year. At Princeton University a fortunate contact took place. Solomon Lefschetz noted the presence of this promising youngster, and encouraged him to take a look at Riemann matrices. These are matrices that arise in the theory of complex manifolds; the main problems concerning them had remained unsolved for more than half a century. The project was perfect for Albert, for it connected closely with the theory of algebras he was so successfully developing. A series of papers ensued, culminating in complete solutions of the outstanding problems concerning Riemann matrices. For this work he received the American Mathematical Society's 1939 Cole prize in algebra.

From 1929 to 1931 he was an instructor at Columbia University. Then the young couple, accompanied by a baby boy less than a year old, happily returned to the University of Chicago. He rose steadily through the ranks: assistant professor in 1931, associate professor in 1937, professor in 1941, chairman of the Department of Mathematics from 1958 to 1962, and dean of the Division of Physical Sciences from 1962 to 1971. In 1960 he received a Distinguished Service Professorship, the highest honor that the University of Chicago can confer on a faculty member; appropriately it bore the name of E. H. Moore, chairman of the Department from its first day until 1927.

The decade of the 1930's saw a creative outburst. Approximately sixty papers flowed from his pen. They covered a wide range of topics in algebra and the theory of numbers beyond those I have mentioned. Somehow, he also found the time to write two important books. *Modern Higher Algebra* (1937) was a widely used textbook—but it is more than a textbook. It remains in print to this day, and on certain subjects it is an indispensable reference. *Structure of Algebras* (1939) was his definitive treatise on algebras and formed the basis for his 1939 Colloquium Lectures to the American Mathematical Society. There have been later books on algebras, but none has replaced *Structure of Algebras*.

The academic year 1933–1934 was again spent in Princeton, this time at the newly founded Institute for Advanced Study. Again, there were fruitful contacts with other mathematicians. Albert has recorded that he found Hermann Weyl's lectures on Lie algebras stimulating. Another thing that happened was that Albert was introduced to Jordan algebras.

The physicist Pascual Jordan had suggested that a certain kind of algebra, inspired by using the operation $xy + yx$ in an associative algebra, might be useful in quantum mechanics. He enlisted von Neumann and Wigner in the enterprise, and in a joint paper they investigated the structure in question. But a crucial point was left unresolved; Albert supplied the missing theorem. The paper appeared in 1934 and was entitled "On a Certain Algebra of Quantum Mechanics." A seed had been planted that Albert was to harvest a decade later.

Let me jump ahead chronologically to finish the story of Jordan algebras. I can add a personal recollection. I arrived in Chicago in early October 1945. Perhaps on my very first day, perhaps a few days later, I was in Albert's office discussing some routine matter. His student Daniel Zelinsky entered. A torrent of words poured out, as Albert told him how he had just cracked the theory of special Jordan algebras. His enthusiasm was delightful and contagious. I got into the act and we had a spirited discussion. It resulted in arousing in me an enduring interest in Jordan algebras.

About a year later, in 1946, his paper appeared. It was followed by "A Structure Theory for Jordan Algebras" (1947) and "A Theory of Power-Associative Commutative Algebras" (1950). These three papers created a whole subject; it was an achievement comparable to his study of Riemann matrices.

World War II brought changes to the Chicago campus. The Manhattan Project took over Eckhart Hall, the mathematics building (the self-sustaining chain reaction of December 1942 took place a block away). Scientists in all disciplines, including mathematics, answered the call to aid the war effort against the Axis. A number of mathematicians assembled in an Applied Mathematics Group at Northwestern University, where Albert served as associate director during 1944 and 1945. At that time, I was a member of a similar group at Columbia, and our first scientific interchange took place. It concerned a mathematical question arising in aerial photography; he gently guided me over the pitfalls I was encountering.

Albert became interested in cryptography. On November 22, 1941, he gave an invited address at a meeting of the American Mathematical Society in Manhattan, Kansas, entitled "Some Mathematical Aspects of Cryptography."* After the war he continued to be active in the fields in which he had become an expert.

In 1942 he published a paper entitled "Non-Associative Algebras." The date of receipt was January 5, 1942, but he had already presented it to the American Mathematical Society on September 5, 1941, and he had lectured on the subject at Princeton and Harvard during March of 1941. It seems fair to name one of these presentations the birth date of the American school of non-associative algebras, which he singlehandedly founded. He was active in it himself for a quarter of a century, and the school continues to flourish.

Albert investigated just about every aspect of non-associative algebras. At times a particular line of attack failed to fulfill the promise it had shown; he would then exercise his sound instinct and good judgment by shifting the assault to a different area. In fact, he repeatedly displayed an uncanny knack for selecting projects which later turned out to be well conceived, as the following three cases illustrate.

* The twenty-nine-page manuscript of this talk was not published, but Chicago's Department of Mathematics has preserved a copy.

(1) In the 1942 paper he introduced the new concept of isotopy. Much later it was found to be exactly what was needed in studying collineations of projective planes.

(2) In a sequence of papers that began in 1952 with "On Non-Associative Division Algebras," he invented and studied *twisted fields*. At the time, one might have thought that this was merely an addition to the list of known non-associative division algebras, a list that was already large. Just a few days before this paragraph was written, Giampaolo Menichetti published a proof that every three-dimensional division algebra over a finite field is either associative or a twisted field, showing conclusively that Albert had hit on a key concept.

(3) In a paper that appeared in 1953, Erwin Kleinfeld classified all simple alternative rings. Vital use was made of two of Albert's papers: "Absolute-Valued Algebraic Algebras" (1949) and "On Simple Alternative Rings" (1952). I remember hearing Kleinfeld exclaim "It's amazing! He proved exactly the right things."

The postwar years were busy ones for the Alberts. Just the job to be done at the University would have absorbed all the energies of a lesser man. Marshall Harvey Stone was lured from Harvard in 1946 to assume the chairmanship of the Mathematics Department. Soon Eckhart Hall was humming, as such world famous mathematicians as Shiing-Shen Chern, Saunders Mac Lane, André Weil, and Antoni Zygmund joined Albert and Stone to make Chicago an exciting center. Albert taught courses at all levels, directed his stream of Ph.D.'s (see the list at the end of this memoir), maintained his own program of research, and helped to guide the Department and the University at large in making wise decisions. Eventually, in 1958, he accepted the challenge of the chairmanship. The main stamp he left on the Department was a project dear to his heart: maintaining a lively flow of visitors and research instructors, for whom he skillfully got support in the form of research grants. The University cooperated by making an apartment building available to house the visitors. Affectionately called "the compound," the modest building has been the birthplace of many a fine theorem. Especially memorable was the academic year 1960–1961, when Walter Feit and John Thompson, visiting for the entire year, made their big breakthrough in finite group theory by proving that all groups of odd order are solvable.

Early in his second three-year term as chairman, Albert was asked to assume the demanding post of dean of the Division of Physical Sciences. He accepted, and served for nine years. The new dean was able to keep his mathematics going. In 1965 he returned to his first love: associative division algebras. His retiring presidential address to the American Mathematical Society, "On Associative Division Algebras," presented the state of the art as of 1968.

Requests for his services from outside the University were widespread and frequent. A full tabulation would be long indeed. Here is a partial list: consultant, Rand Corporation; consultant, National Security Agency; trustee, Institute for Advanced Study; trustee, Institute for Defense Analyses, 1969–1972, and director of its Communications Research Division, 1961–1962; chairman, Division of Mathematics of the National Research Council, 1952–1955; chairman, Mathematics Section of the National Academy of Sciences, 1958–1961; chairman, Survey of Training and Research Potential in the Mathematical Sciences, 1955–1957 (widely known as the "Albert Survey"); president, American Mathematical Society, 1965–1966; participant and then director of Project SCAMP at the University of California at Los Angeles; director, Project ALP (nicknamed "Adrian's little project"); director, Summer 1957 Mathematical Conference at Bowdoin College, a project of the Air Force Cambridge Research Center; vice-president, International Mathematical Union; and delegate, IMU Moscow Symposium, 1971, honoring

Vinogradov's eightieth birthday (this was the last major meeting he attended).

Albert's election to the National Academy of Sciences came in 1943, when he was thirty-seven. Other honors followed. Honorary degrees were awarded by Notre Dame in 1965, by Yeshiva University in 1968, and by the University of Illinois Chicago Circle Campus in 1971. He was elected to membership in the Brazilian Academy of Sciences (1952) and the Argentine Academy of Sciences (1963).

In the fall of 1971, he was welcomed back to the third floor of Eckhart Hall (the dean's office was on the first floor). He resumed the role of a faculty member with a zest that suggested that it was 1931 all over again. But as the academic year 1971–1972 wore on, his colleagues and friends were saddened to see that his health was failing. Death came on June 6, 1972. A paper published posthumously in 1972 was a fitting coda to a life unselfishly devoted to the welfare of mathematics and mathematicians.

In 1976 the Department of Mathematics inaugurated an annual event entitled the Adrian Albert Memorial Lectures. The first lecturer was his long-time colleague Professor Nathan Jacobson of Yale University.

MRS. FRIEDA ALBERT was generous in her advice concerning the preparation of this memoir. I was also fortunate to have available three previous biographical accounts. "Abraham Adrian Albert, 1905–1972," by Nathan Jacobson (*Bull. Am. Math. Soc.*, 80: 1075–1100), presented a detailed technical appraisal of Albert's mathematics, in addition to a biography and a comprehensive bibliography. I also wish to thank Daniel Zelinsky, author of "A. A. Albert" (*Am. Math. Mon.*, 80:661–65), and the contributors to volume 29 of *Scripta Mathematica*, originally planned as a collection of papers honoring Adrian Albert on his sixty-fifth birthday. By the time it appeared in 1973, the editors had the sad task of changing it into a memorial volume; the three-page biographical sketch was written by I. N. Herstein.

PH. D. STUDENTS OF A. A. ALBERT

1934 ANTOINETTE KILLEN: The integral bases of all quartic fields with a group of order eight.
OSWALD SAGEN: The integers represented by sets of positive ternary quadratic non-classic forms.

1936 DANIEL DRIBIN: Representation of binary forms by sets of ternary forms.

1937 HARRIET REES: Ideals in cubic and certain quartic fields.

1938 FANNIE BOYCE: Certain types of nilpotent algebras.
SAM PERLIS: Maximal orders in rational cyclic algebras of composite degree.
LEONARD TORNHEIM: Integral sets of quaternion algebras over a function field.

1940 ALBERT NEUHAUS: Products of normal semi-fields.

1941 FRANK MARTIN: Integral domains in quartic fields.
ANATOL RAPOPORT: Construction of non-Abelian fields with prescribed arithmetic.

1942 GERHARD KALISCH: On special Jordan algebras.
RICHARD SCHAFER: Alternative algebras over an arbitrary field.

1943 ROY DUBISCH: Composition of quadratic forms.

1946 DANIEL ZELINSKY: Integral sets of quasiquaternion algebras.

1950 NATHAN DIVINSKY: Power associativity and crossed extension algebras.
CHARLES PRICE: Jordan division algebras and their arithmetics.

1951 MURRAY GERSTENHABER: Rings of derivations.
DAVID MERRIEL: On almost alternative flexible algebras.
LOUIS WEINER: Lie admissible algebras.

1952 LOUIS KOKORIS: New results on power-associative algebras.
JOHN MOORE: Primary central division algebras.

1954 ROBERT OEHMKE: A class of non-commutative power-associative algebras.
EUGENE PAIGE: Jordan algebras of characteristic two.

1956 RICHARD BLOCK: New simple Lie algebras of prime characteristic.

1957 JAMES OSBORN: Commutative diassociative loops.

1959 LAURENCE HARPER: Some properties of partially stable algebras.

1961 REUBEN SANDLER: Autotopism groups of some finite nonassociative algebras.
PETER STANEK: Two element generation of the symplectic group.

1964 ROBERT BROWN: Lie algebras of types E_6 and E_7.

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