Carolyn Eisele, Emeritus Professor of Mathematics at Hunter College since 1972, earned an A.M. from Columbia University in 1925. She is a lecturer and writer of international renown on the history and philosophy of mathematics and science of the late 19th and early 20th centuries, as well on new elements of mathematics. Specializing in the thought of Charles S. Peirce, about whom she has edited several books, Professor Eisele has had a major role in the Charles S. Peirce Society. In addition, she has served on the advisory committee of the Fulbright and Smith-Mundt Awards, is a fellow of the New York Academy of Sciences and is a member of the Mathematical Association of America.

Thomas S. Fiske and Charles S. Peirce

CAROLYN EISELE

The *Bulletin of the American Mathematical Society*, volume XI, February 1905 carried the Presidential address of Thomas Scott Fiske telling of "Mathematical Progress in America." It had been delivered before the Society at its eleventh annual meeting on December 29, 1904. Fiske spoke of three periods during which one might consider the development of pure mathematics on the American scene. He pointed in the first period to the work of Adrain and of Bowditch and of Benjamin Peirce whose *Linear Associative Algebra* was the first important research to come out of America in the field of pure mathematics. The second period, Fiske said, began with the arrival in 1876 of James Joseph Sylvester (1814–1897) to occupy the Chair of Mathematics at the newly founded Johns Hopkins University in Baltimore where his first class consisted of G. B. Halsted. C. S. Peirce, son of Benjamin, once described Sylvester as "perhaps the mind most exuberant in ideas of pure mathematics of any since Gauss." Shortly after Sylvester's arrival in Baltimore, he established the *American Journal of Mathematics* (*AJM*), a great stimulant to original mathematical research, which ultimately included some thirty papers by himself. In 1883 he returned to England to become Savilian Professor of Geometry at Oxford.

The first ten volumes of the *AJM* contained papers by about ninety different writers and covered the years 1878 to 1888. About thirty were mathematicians of foreign countries; another twenty were pupils of Professor Sylvester;

others had come under the influence of Benjamin Peirce; some had been students at German Universities; some were in large degree self-trained. Several of them had already sent papers abroad for publication in foreign journals. Among the contributors to the early volumes of the *American Journal of Mathematics* special mention must be made of Newcomb, Hill, Gibbs, C. S. Peirce, McClintock, Johnson, Story, Stringham, Craig, and Franklin.

Reminiscing in 1939, Fiske recalled in the *Bulletin* that while a graduate student in the Department of Mathematics at Columbia University, he had been urged in 1887 by Professor Van Amringe to visit Cambridge, England. There he attended all the mathematical lectures in which he was interested and met Cayley. On November 24, 1888, after his return to New York, Fiske interested two fellow students, Jacoby and Stabler, in the idea of creating a local mathematical society which was to meet monthly for the purpose of discussing mathematical topics. Thus began the New York Mathematical Society and the third period in the Fiske account of American mathematical progress.

By December 1890, the idea of the Society publishing a *Bulletin*, as did the London group, was suggested and led to the decision to have the American Society ape them. The first number appeared in 1891 with Fiske as editor-in-chief. By 1899 arrangements were made for the publication of a *Transactions* and Fiske again became a member of the editorial staff (1899–1905).[1]

As secretary of the Society, Fiske corresponded with a few of the top mathematicians of this country to determine if their assistance could be counted on for the new enterprise. Cooperating at once were Newcomb, Johnson, Craig, and Fine.

Among those invited in 1891 were two unusual individuals: Charles P. Steinmetz and C. S. Peirce. Fiske relates that Steinmetz who was born in Breslau on April 9, 1865, was "a student at the University of Breslau where he had been the ablest pupil of Professor Heinrich Schroeter. In the spring of 1888 he was about to receive the degree of Ph.D., but in order to escape arrest as a socialist he was compelled to flee to Switzerland. Thence he made his way to America, arriving in New York June 1, 1889." Fiske continued, "About a year later my attention was attracted to an article of sixty pages or more in the *Zeitschrift für Mathematik und Physik* on involutory correspondences defined by a three-dimensional linear system of surfaces of the *n*th order by Charles Steinmetz of New York. This was the doctor's dissertation."

Fiske tells of having invited him to Columbia University and of having offered help with the English in his papers. He also invited him to become a

[1]We learn in an article by Albert E. Meder, Jr., in *Science* (September 9, 1938) that the Society had come to feel the need of an outlet for publication of *original* mathematical research. An early hope that a cooperative arrangement could be worked out with the *American Journal of Mathematics*, then prospering at the Johns Hopkins University, was soon dashed and the separate *Transactions* came into being.

member of the New York Mathematical Society where he presented a number of papers, two of which were published in the *American Journal of Mathematics*. Although soon gaining fame as one of America's leading electrical engineers, he remained a member until his death on October 26, 1923.

The other addition to the Society in 1891 is of particular interest to us. "Charles S. Peirce, B. Sc., M.A., member of the National Academy of Sciences" was elected at the November meeting. It is of interest to learn that Van Amringe had proposed Peirce's name for membership and that Harold Jacoby had sent the invitation. It is to be noted that Peirce was already a member of the London Mathematical Society. The Fiske *Bulletin* account of of 1939 tells the story this way:

> Conspicuous among those who in the early nineties attended the monthly meetings in Professor Van Amringe's lecture room was the famous logician, Charles S. Peirce. His dramatic manner, his reckless disregard of accuracy in what he termed "unimportant details," his clever newspaper articles describing the meetings of our young Society interested and amused us all. He was advisor of the New York Public Library for the purchase of scientific books and writer of the mathematical definitions in the Century Dictionary. He was always hard up, living partly on what he could borrow from friends, and partly from odd jobs such as writing book reviews for the *Nation* and the *Evening Post*.... At one meeting of the Society, in an eloquent outburst on the nature of mathematics C. S. Peirce proclaimed that the intellectual powers essential to the mathematician were "concentration, imagination, and generalization." Then, after a dramatic pause, he cried: "Did I hear someone say demonstration? Why, my friends," he continued, "demonstration is merely the pavement upon which the chariot of the mathematician rolls."

The last sentence in the Fiske account causes one to pause momentarily and observe that Fiske was apparently more sensitive to the nature of Peirce's vast outpourings in those days than most of their contemporaries in mathematics could be. Truly Peirce had elected to trace the taming of the human mind in its ordered discovery of the patterns of nature's design to the development of logical restraint. Peirce's probes for materials in his analytical development of such themes led to massive descriptions of events in the history of mathematics and science. And his attempts to systematize those historical reflections permeate Peirce's writings generally — philosophical, mathematical, scientific.[2]

[2]Fortunately his attempts to gather together such data for a volume are still extant and have been recently collected into a two-volume study by the present writer with the title *Historical Perspectives on Peirce's Logic of Science: A History of Science* (Mouton, 1985).

A few samples of C. S. Peirce's productivity in mathematics may be found in the early pages of the *American Journal of Mathematics*, and serve to give some idea of his involvement in that creative operation. He was officially a member of the Mathematics Department at the Johns Hopkins University under Sylvester while still a member of the Coast Survey. His contributions to the AJM were

Review of *Esposizione del Metodo dei Minimi Quadrati* per Annibale Ferrero. 1878 1:59–63

"On the Ghosts in Rutherfurd's Diffraction Spectra." 1879 2:330–347

"A Quincuncial Projection of the Sphere." 1879 2:394–396

"On the Algebra of Logic." 1880 3:15–57

"On the Logic of Numbers." 1881 4:85–95

Linear Associative Algebra by Benjamin Peirce with Notes and Addenda by C. S. Peirce. 1881 4:97–229

"On the Algebra of Logic: A Contribution to the Philosophy of Notation." 1885 7:180–202

The history surrounding these publications provides the earliest evidence of Peirce's interest and competence in mathematics and mathematical exposition. It also suggests something of the wide range spanned by the mathematical interests of the time. It will be instructive to take two examples.

The first is Charles's preparation of his father's *Linear Associative Algebra* for publication by Van Nostrand in 1882. The title page of the separate edition describes Benjamin Peirce as "Late Professor of Astronomy and Mathematics in Harvard University and Superintendent of the United States Coast Survey" and tells that this is a new edition, with addenda and notes by C. S. Peirce, son of the author. One can foresee in Charles's editing the early rivulet of an intellectual orientation that would swell into a torrential force by the end of the century to color his thought in every area. Lithographed copies of the text had been distributed by Benjamin in 1870. The new book contains "separate copies extracted from the *American Journal of Mathematics*." On page one is found the famous Benjamin Peirce opening statement: "Mathematics is the science which draws necessary conclusions." Appended to the volume in this form are a reprint of a paper by Benjamin (1875), "On the Uses and Transformations of Linear Algebra" which had been presented to the American Academy of Arts and Sciences, May 11, 1875, and two papers by Charles, one "On the Relative Forms of the Algebras," the other "On the Algebras in which Division is Unambiguous."

A full list of Benjamin Peirce's writings[3] serves to remind us again that in an earlier age, not yet benefitting from modern technical skills, America already possessed a reserve of great intellectual talent, mathematical and

[3]Given by Raymond Clare Archibald in *Benjamin Peirce* (Math. Assoc. of America, 1925), in the biographical sketch.

otherwise. The titles of the papers in that list give some awareness of C. S. Peirce's general mathematical exposure in this early period before and during his association with Fiske.[4]

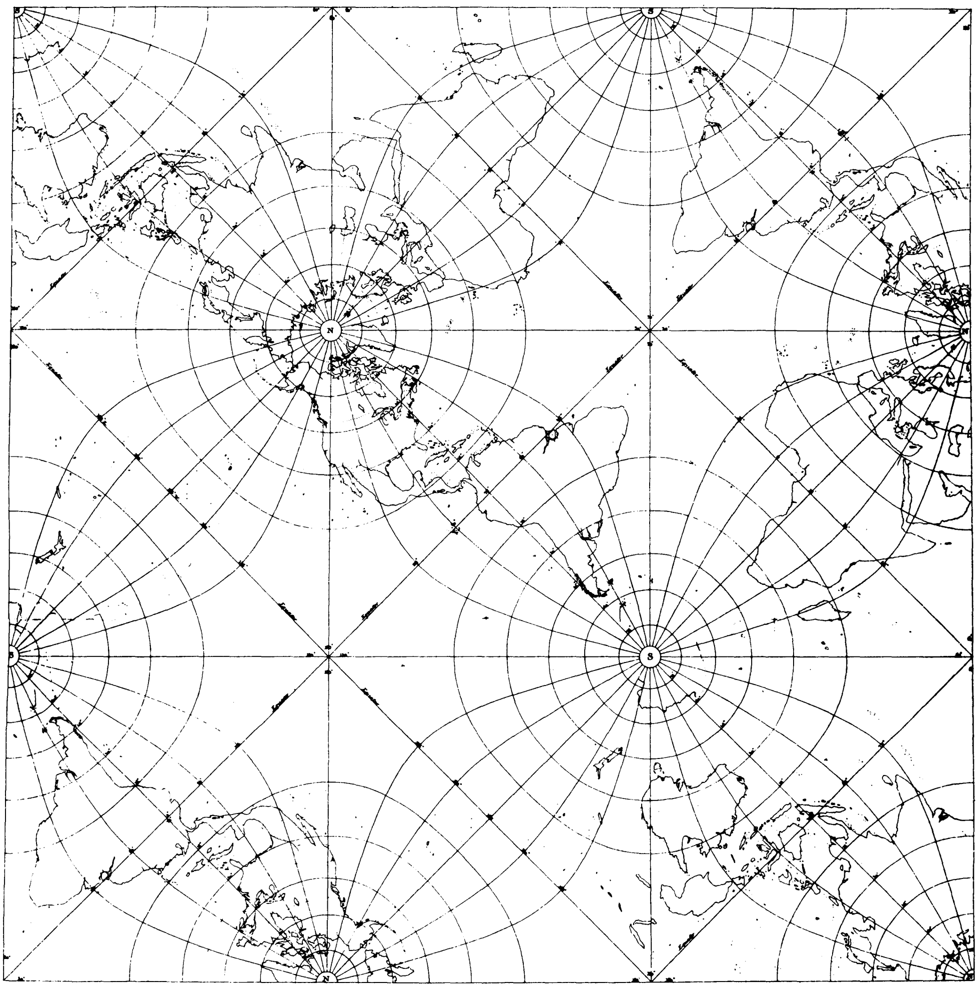
The second example pertains to his map project. In Peirce's report to Superintendent Patterson of the Coast and Geodetic Survey on July 17, 1879, the section on pendulum observation states that "the theory of conform map projection has been studied with reference to its use in the study of gravity; and a new projection has been invented." A year earlier Peirce had been invited by Sylvester to publish a paper on map projections in the forthcoming issue of the *American Journal of Mathematics.* And at the meetings of the National Academy of Sciences (April 15–18, 1879) Peirce presented a paper "On the projections of the sphere which preserve the angles." Moreover the report to Superintendent Patterson showing the progress of work during the fiscal year ending June, 1879, describes Peirce's invention as follows:

> Among several forms of projection devised by Assistant Peirce, there is one by which the whole sphere is represented upon repeating squares. This projection, as showing the connection of all parts of the surface, is convenient for meteorological, magnetological, and other purposes. The angular relation of meridians and parallels is strictly preserved; and the distortion of areas is much short of the distortion incidental to any other projection of the entire sphere.

Peirce himself described it later in the Johns Hopkins University Circulars as possessing the following qualities: The whole sphere is represented on repeating squares; the part where the exaggeration of scale amounts to double that at the center is only 9 percent of the area of the sphere as against 13 percent for Mercator's projection, and 50 percent for the Stereographic; the angles are exactly preserved; the curvature of lines representing great circles is, in every case, very slight over the greater part of their length.

The map was published not only in the second volume of the *American Journal of Mathematics* in 1879 but also in 1882 by Thomas Craig of the Coast Survey and a member of the Mathematics Department at the Johns Hopkins University. He included it in his Coast Survey publication, *A Treatise on Projection*, August 19, 1880, and listed Peirce as one of the greatest map-makers to date. It had also appeared in Appendix #15 of the Coast Survey report for 1877, and yet Peirce's invention at that time was not adopted for use by any government or private agency. Max Fisch has mentioned also a meeting of the Metrological Society at Columbia University on May 20, 1879, and inclusion in the report of the Committee on Standard Time of an

[4]A full account of C. S. Peirce's many mathematical interests can be found in the *New Elements of Mathematics* (5 books; 4 volumes, Mouton, 1976), as edited by C. Eisele.



The Quincuncial Map Projection invented by C. S. Peirce in 1876.

(From the Charles S. Peirce Collection in the Houghton Library, Harvard University.)

appendix on "a quincuncial projection of the world," prepared by Mr. C. S. Peirce.

Some sixteen years after its appearance in the *American Journal of Mathematics* the map came to the attention of the noted mathematician James P. Pierpont who considered Peirce's work a very elegant representation of the sphere upon the plane. Pierpont had found a slight error in Peirce's algebraic treatment and published a "Note on C. S. Peirce's Paper on a Quincuncial Projection of the Sphere." A note from Pierpont to Peirce (December 10, 1894) asked if Peirce had ever published other papers on the projection. "In case you have, I should be greatly obliged to you if you would refer me to them."

Peirce described his projection in the following terms:

> For meteorological, magnetological and other purposes, it is convenient to have a projection of the sphere which shall show the connection of all parts of the surface. This is done by the one shown in the plate. It is an orthomorphic or conform projection formed by transforming the stereographic projection, with a pole at infinity, by means of an elliptic function. For that purpose, l being the latitude, and θ the longitude, we put
>
> $$\cos^2\phi = \frac{\sqrt{1-\cos^2 l\cos^2\theta} - \sin l}{1+\sqrt{1-\cos^2 l\cos^2\theta}},$$
>
> and then $\frac{1}{2}F\phi$ is the value of one of the rectangular coordinates of the point on the new projection. This is the same as taking
>
> $$\begin{aligned}&\cos am(x+y\sqrt{-1})(\text{angle of mod.} = 45^\circ)\\ &\quad= \tan\tfrac{p}{2}(\cos\theta + \sin\theta\sqrt{-1}),\end{aligned}$$
>
> where x and y are the coordinates on the new projection, p is the north polar distance. A table of these coordinates is subjoined.
>
> Upon an orthomorphic potential the parallels represent equipotential or level lines for the logarithmic projection, while the meridians are the lines of force. Consequently we may draw these lines by the method used by Maxwell in his *Electricity and Magnetism* for drawing the corresponding lines for the Newtonian potential. That is to say, let two such projections be drawn upon the same sheet, so that upon both are shown the same meridians at equal angular distances, and the same parallels at such distances that the ratio of successive values of $\tan\frac{p}{2}$ is constant. Then, number the meridians and also the parallels. Then draw curves through the intersections of meridians with meridians, the sums of numbers of the interesecting meridians being constant on any one curve. Also, do the same thing for the parallels. Then these curves will

represent the meridians and parallels of a new projection having north poles and south poles wherever the component projections had such poles.

Functions may, of course, be classified according to the pattern of the projection produced by such a transformation of the stereographic projection with a pole at the tangent points. Thus we shall have—

1. Functions with a finite number of zeros and infinities (algebraic functions).

2. Striped functions (trigonometric functions). In these the stripes may be equal, or may vary progressively, or periodically. The stripes may be simple, or themselves compounded of stripes. Thus, $\sin(a \sin z)$ will be composed of stripes each consisting of a bundle of parallel stripes (infinite in number) folded over onto itself.

3. Chequered functions (elliptic functions).

4. Functions whose patterns are central or spiral.

The description was accompanied by two tables for the construction of the map:

I. Table of Rectangular Coordinates for Construction of the "Quincuncial Projection";

II. Preceding Table Enlarged for the Spaces Surrounding Infinite Points.

It is of still further interest to note that in the January–March 1946 edition of *Surveying and Mapping*, Albert A. Stanley, Special Assistant to the Director, Coast and Geodetic Survey, published an article in which he explained that

> The U.S. Coast and Geodetic Survey recently published Chart # 3092, showing major International Air Routes on a chart constructed on a Quincuncial Projection of the sphere.... The resulting configuration of land areas is conformal with the whole sphere being represented on repeating squares; also the major air routes, which for the most part follow approximate great circles, are in areas of least distortion, and in most cases are shown as straight lines. Angles of intersection are preserved exactly, and the maximum exaggeration of scale is less on the quincuncial projection than on either the Mercator or the stereographic projection.

Stanley attributes the invention of the chart to Peirce. Its revival was due to its showing "the major air routes as approximate straight lines on a world outline preserving satisfactory shapes...."

By 1894, after the Society had become the American Mathematical Society, Fiske was writing to Peirce suggesting that he share with them knowledge exhibited in articles in the *American Journal of Mathematics* and in his notable work for the *Century Dictionary* (1889) where he is described in the "List of Collaborators" as Lecturer on Logic at the Johns Hopkins University and as a member of the U.S. Coast and Geodetic Survey. The Dictionary terms for which he was responsible were "Logic; Metaphysics; Mathematics; Mechanics; Astronomy; Weights and Measures."

Peirce's scientific and mathematical writings reflected his many different areas of interest. For example, one must wonder if Fiske was aware of Peirce's early involvement in mathematical economics when he himself wrote an article review of Irving Fischer's *Mathematical Investigations in the Theories of Value and Price* in the *Bulletin* of 1893. Indeed, Max Fisch, the eminent Peirce scholar, has called this writer's attention to a letter Peirce sent to his father from the U.S. Coast Survey Office in Washington on December 19, 1871.

> There is one point on which I get a different result from Cournot, and it makes me suspect the truth of the proposition that the seller puts his price so as to make his profits a maximum. Suppose two sellers only of the same sort of article which costs them nothing. Then
>
> $$y_1 + D_{x_1} y_1 \cdot x_1 = 0$$
> $$y_2 + D_{x_2} y_2 \cdot x_2 = 0$$
>
> Now each seller must reflect that if he puts down his price the other will put down his just as much, so that if y is the total sales $\frac{y_1}{y}$ is constant. Then the competition won't alter the price. But suppose the seller does *not* make this reflection. Then if his price is lower than the other man's he gets all his customers besides attracting new ones. Then the price is forced down to zero, by the successive action of the two sellers for $D_{x_1} y_1 = -\infty$. Cournot's result is
>
> $$y + 2D_x y \cdot x = 0.$$
>
> This is as though each seller in putting down his price expected to get all the new customers and yet didn't expect to get away any of the other seller's customers. Which is not consistent. He reasons in this way. The prices asked by the two sellers are the same. Put
>
> $$x = f(y_1 + y_2).$$
>
> Then to make xy_1 and xy_2 maxima we have
>
> $$f(y_1 + y_2) + y_1 f'(y_1 + y_2) = 0$$
> $$f(y_1 + y_2) + y_2 f'(y_1 + y_2) = 0$$

Summing, etc., he gets his result. But his f' is indefinite. If it is a previous condition that the prices are the same

$$D_{y_1} f(y_1 + y_2) = 2D_y f(y_1 + y_2).$$

Am I not right? He seems to think there is some magic in considering $D_y x$ instead of $D_x y$. According to me the equation is

$$y + xD_x y - x(D_{x_1} y_2 + D_{x_2} y_1) = 0$$

if the sellers reckon on the other's price *not* changing and is $y + xD_x y = 0$ if they consider this change.

P.S. What puzzles me is this. The sellers must reckon (if they are not numerous) on all following any lowering of the price. Then, if you take into account the cost, competition would generally (in those cases) *raise* the price by raising the final cost. But is not this a *fact*?

This letter is closely associated with another of similar content sent to Simon Newcomb, a greatly celebrated scientist and mathematician of that period. Indeed Newcomb was to be elected for the years 1897 and 1898 to the office of the presidency of the American Mathematical Society. Many years before that he had studied mathematics at the Lawrence Scientific School with Benjamin Peirce. Newcomb as well as C. S. Peirce became members of the National Academy of Sciences, Newcomb in 1869, Peirce in 1877.[5] The Peirce letter to Newcomb follows.[6]

Dec. 17, 1871

U.S. Coast Survey Office
Washington

Dear Sir,

What I mean by saying that the law of Supply and Demand only holds for unlimited competition is this. I take the law to be, that the price of an article will be such that the amount the producers can supply at that price with the greatest total profit, is equal to

[5] An intensive study of their similar-dissimilar lives is to be found in the "Charles S. Peirce-Simon Newcomb Correspondence" by this writer in the *Proceedings of the American Philosophical Society*, **101** #5, October 1957, 409–433. This paper is reprinted in *Studies in the Scientific and Mathematical Philosophy of Charles S. Peirce. Essays by Carolyn Eisele.* Edited by R. M. Martin (Mouton, 1979). For elaboration on several of the topics discussed here see Eisele, 1979.

[6] Peirce's interest in the field of Economics has been documented and well recognized by Baumol and Goldfeld, the noted economists, in their *Precursors in Mathematical Economics* (London School of Economics and Political Science, 1968).

what the consumers will take at that price. This is the case with unlimited competition because nothing that any individual producer does will have an appreciable effect on the price; therefore he simply produces as much as he can profitably. But when production is not thus stimulated, the price will be higher and at that higher price a greater amount might be profitably supplied.

To state this algebraically:—

The amount that can be profitably produced at a certain price X is the value of y which makes certain price X is the value of y which makes $(X_y - z)$ maximum so that $X - D_y z = 0$. But the price x which the producer will set will be that which will make $(xy - z)$ a maximum so that $y + D_x y(x - D_y z) = 0$.

Clearly $x > X$ because $D_x y < 0$. In the case of unlimited competition, however, the price is not at all influenced by any single producer so that x is constant $D_x y = \infty$ and then the second equation reduces to the first. If this differentiation by a constant seems outlandish, you can get the same result another way. But it is right, for if the producer, in this case, lowers his price below what is best for him there will be an immense run upon him, if he raises it above that he will have no sales at all, so that $D_x y = -\infty$.

If the law of demand and supply is stated as meaning that no more will be produced than can be sold, then it shows the limitation of production, but is not a law regulating the price.

Yours very truly,

C. S. Peirce

Prof. Simon Newcomb U.S.N.

Observatory

P.S. This is all in Cournot

In his outline of remarks for the introductory lecture on the study of logic at the Johns Hopkins University (September 1882), Peirce declared that the scientific specialists — pendulum swingers and the like — are doing a great and useful work... "but the higher places in science in the coming years are for those who succeed in adapting the methods of one science to the investigation of another. That is what the greatest progress of the passing generation consists in." Peirce spoke of Cournot adapting to political economy the calculus of variations. He himself had so adapted the calculus of variations in his "Note on the Theory of Economy of Research" in the *Report of the Superintendent of the United States Coast Survey* for the Fiscal Year ending with June 1876.

Since the *Bulletin* was devoted primarily to historical and critical articles, and since Peirce was knowledgeable on these matters, he was besought by

Fiske on February 22, 1894, to write an article on "English Mathematical Nomenclature." On April 6, 1894, Fiske expressed his pleasure that Peirce was to attend the meeting on April 8 and give an account of the *Arithmetic* of Rollandus. He asked if Peirce would "throw remarks into a form for publication in the *Bulletin*." Peirce did throw such remarks into a form that he used in a report of the meeting of the Society for the *New York Times* of April 8, 1894. It now follows:

> At a meeting of the New York Mathematical Society, held yesterday afternoon in Hamilton Hall, Columbia College, two very interesting papers on mathematical subjects were presented. The first, read by Prof. Woolsey Johnson, was entitled, "Gravitation and Absolute Units of Force".... C. S. Peirce exhibited the manuscript of an extensive work on arithmetic from the valuable collection of George A. Plimpton. It was written in Latin in the year 1424, and has been entirely unknown to the historians of mathematics. It was written at the command of John of Lancaster, Duke of Bedford, son of Henry IV of England, who, after the death of his brother, Henry V, and during the minority of his nephew, Henry VI, was made Protector of England and Regent of France. He was not only one of the most sagacious and virtuous of rulers, but also a man of learning. In August, 1423, was issued an ordinance for the restoration of studies in the University of Paris, which had shrunk almost to nothing during the wars. It was particularly desired that the studies of logic and of theology, which had been almost exclusively cultivated at the celebrated seat of learning, should make way for a certain amount of mathematics. The preparation of this textbook of arithmetic, now brought to light, was intrusted to a certain Portuguese physician, Rollandus, who was a minor canon of the Sainte Chapelle, which every tourist remembers, opposite the Louvre. It is evident from the words of the dedication that the work must have been completed before the great battle of Ivry, August 14, 1424. John himself, according to the preface, took an active part in the preparation of the book, directing what should be included and what excluded. The result was a treatise much superior to any other of the same age; but it was probably found too difficult for the students, since it has fallen completely into oblivion, so much so that not a single copy except that of Mr. Plimpton has ever been brought to notice. The dedication is something of a curiosity.

Later that year the *Bulletin* notes that on November 24, 1894, Peirce read a second paper entitled "Rough Notes on Geometry, Constitution of Real Space." Now it happens that among the many Peirce mathematical manuscripts in the Peirce Collection at Harvard University are several particularly worthy of notice. For example there is an incomplete letter, page one missing, but signed by Peirce and, more important for this essay, originally folded with Fiske's name written in Peirce's hand on the back of the last page of manuscript (ms 121). Since no other mathematics manuscript so well warrants the above descriptive title, it is to be assumed that this letter contained the essence of what was read at the Society meeting on November 24, 1894. References in the opening paragraph on the top of page 2 lead one to surmise that Peirce had been presenting the Bessel-Gaussian discussion on the nature of space. Bessel apparently had made it clear that

> the *strict geometrical truth* is that the sum of the three angles of a triangle is 180° *minus* a hypothetical correction. (He should have said a constant proportion of the area.) He adds that the Euclidean geometry is practically true, at least for terrestrial figures. The limitation is significant, considering that the business of Bessel's life was with nonterrestrial figures. Gauss, after fourteen months' interval, expresses his delight at learning that Bessel had come to that conclusion. The certainty of arithmetic and algebra, he adds, is absolute; but then they do not relate to anything but our own creations (unseres Geistes Product). But space has a reality without us (the Newtonian, anti-Leibnizian, anti-Kantian doctrine), and its laws cannot be known a priori. In other words, Gauss divides Geometry into Physical Geometry, a branch of physics which inquires what the properties of real space are, and all whose conclusions are affected with a probable error, and Mathematical Geometry, which in consequence of the suggestions of physical geometry, develops certain ideas of space. I do not merely say to trace out the consequences of hypotheses for certainly Gauss would have held that it was part of the mathematician's function to invent the n-dimensional and other varieties of the space-idea.

Peirce proceeds to point to two fallacies in the reasoning of the first book of Euclid and has need to refer to two basic elements overlooked by Saccheri and Stäckel. He has need to mention elements at infinity and to ascribe the fallacies implied to "arise from Euclid's confidence in his 8th axiom, that the whole is greater than its part.... It has been abundantly shown by Cauchy, G. Cantor, and others that the whole is only necessarily greater than its part when that part is finite and positive.... I maintain that Euclid was himself a non-Euclidean geometer...." Space forbids the inclusion here of the full discussion in the Peirce letter. More evidence of Peirce's flirtation with ideas of n-dimensional space and of infinite elements is to be found in the aforementioned "The Charles S. Peirce–Simon Newcomb Correspondence."

It must come as a surprise to notice his extreme technical skill in applied math. But then the thought of the Peirce who was operating as a technician in the government service is being revealed only now. There is also the matter of his having been assigned to the supervision of pendulum experiments in the Survey in 1872 and, in this connection, to investigate the law of deviation of the plumb line and of the azimuth from the spheroidal theory of the earth's figure. In a review of the work of the Survey in 1909, the United States Department of Commerce and Labor described triangular and astronomical observations connected with it as furnishing the most reliable data for the determination of the figure of the earth that have been contributed by any nation. Moreover the first experimental proof of the flattening of the earth was obtained by a change in the rate of a clock in different latitudes. The change in the oscillation of a pendulum with latitude becomes one of the recognized methods of determining the compression of the earth.

While at the Coast Survey Peirce was, for a short while, in charge of the Office of Weights and Measures. He addressed a meeting of the Society of Weights and Measures at Columbia University on December 30, 1884, on his pendulum experiments and changes in the weight of the Troy and avoirdupois pounds over the years. The report of this meeting provides an instance of Peirce's vast knowledge of the historical, social, and technical aspects of weights and measures.

Further evidence of Peirce's ingenuity must be noted in this day of machine-dependency. Since the American Mathematical Society celebrates its centennial year when the all-powerful role of the computer is well acknowledged, it is of interest to review an early episode in its formative years. In a paper entitled "Logical Machines" which appeared in the *American Journal of Psychology* in 1887, Peirce compares the instruments invented by Jevons and Marquand, and is led to believe that

> Mr. Marquand's machine is a vastly more clear-headed contrivance than that of Jevons.... The secret of all reasoning machines is after all very simple. It is that whatever relation among the objects reasoned about is destined to be the hinge of a ratiocination, that same general relation must be capable of being introduced.... When we perform a reasoning in our unaided minds we do substantially the same thing, that is to say, we construct an image in our fancy under certain general conditions, and observe the result. A piece of apparatus for performing a physical or chemical experiment is also a reasoning machine, insomuch as there are certain relations between its parts, which relations involve other relations that were not expressly intended....I do not think there would be any great difficulty in constructing a machine which should work the logic of relations with a large number of terms. But owing to the great variety of ways in which the same premisses can be combined to produce different conclusions in that branch of logic, the

> machine, in its first state of development, would be no more mechanical than a hand-loom for weaving in many colors with many shuttles. The study of how to pass from such a machine as that to one corresponding to a Jacquard loom, would be very likely to do very much for the improvement of logic.

Another matter of current interest that also attracted the attention of Peirce and his contemporaries was the subject of the four-color problem. In *Graph Theory, 1736–1976*, by Biggs, Lloyd, and Wilson (Clarendon Press-Oxford, 1976), the authors write of the part played in the early history of the four-color problem by the mathematicians of The Johns Hopkins University, Peirce, Story, and Sylvester, and the continuing American interest as evidenced in a talk given by Peirce to the National Academy of Sciences in November 1899. The authors ascribe further developments to "the emerging mathematical might of America."

These examples merely suggest the traffic in mathematical ideas on the American scene in the nineteenth century. In R. C. Archibald's account of the History of the American Mathematical Society in January 1939 he challenges Maxime Bôcher's remark of 1905 that "there is no American mathematics worth speaking of as yet." Archibald wrote: "If facts now common property had been known to Bôcher, he might well have cast his statement into different form."

At that time, even as is true today, Archibald found it necessary to stress the fact that "in the United States before the founding of our Society there was an inspiring mathematical center, and that a considerable body of mathematical work, some of it of first importance, had already been achieved. Many people scattered over the country were then engaged in mathematical pursuits, and among them were not a few who had obtained their doctor's degrees either here or in Europe. Hence the time was opportune for drawing them together" into a Society — on Thanksgiving Day 1888.

This lengthy memorandum is in part not only a tribute to the mathematical and logical ingenuity of one C. S. Peirce but to that most enthusiatic promoter of the cause of mathematics, Thomas S. Fiske. Since the last quarter century has been devoted to adding mathematical and scientific brushstrokes to the Peirce portrait, the writer now recalls with great satisfaction the impact of the Fiske lectures in a graduate course at Columbia University in the mid-twenties. In those days Columbia University was not granting doctorates in mathematics to women. In the course of sympathizing with my friend Helen and myself and advising us on a future course of action, Professor Fiske walked us merrily down Broadway to 42nd Street one evening after class, all the while plying us with fruit from the stands along the line in those different times. How much one might have learned from him about the Peirce who was to absorb so much of the writer's attention so very many years later!