

2020 Who Wants to Be a Mathematician Round Two Qualifying Test

1. For how many prime numbers $p < 300$ is $p^3 - 2p^2$ a positive perfect square (square of a positive integer)? _____
2. Compute $\cos\left(\tan^{-1}\left(\frac{-\sqrt{7}}{3}\right)\right)$ (where \tan^{-1} denotes the inverse tangent function). _____
3. What proportion of the nine-digit numbers that can be formed by permuting the digits of 123456789 are divisible by 36? (Write your answer as a fraction.) _____
4. Let $f(x) = (2x + 3)^3$ and $g(x) = x^3 + x^2 - x - 1$. What is the sum of the coefficients of the polynomial $f(g(x))$? _____
5. The *incircle* (or *inscribed circle*) of a triangle is the circle that is tangent to all three sides of the triangle. What is the radius of the incircle of a triangle with side lengths 13, 14, and 15? _____
6. The integers 3, 4, 5, 6, 12, and 13 are arranged, without repetition, in a horizontal row so that the sum of any two numbers in adjacent positions is a perfect square (square of a positive integer). What is the sum of the first and last numbers in the arrangement? _____
7. Which of the following numbers is the product of three consecutive prime numbers? (circle one)
a. 1223 b. 1309 c. 1989 d. 2431 e. 2717
8. What is the sum of all real solutions of $\sqrt{x + 15 - 8\sqrt{x - 1}} = 2$? _____
9. A jar contains ten balls, numbered 1 to 10. Three balls are randomly drawn from the jar without replacement. What is the probability that no two of the three balls are labeled with consecutive integers? _____
10. (Tie-breaking question) What positive integer is closest to $(e + 2\pi)^{9/2}$? _____