2020 Who Wants to Be a Mathematician Round One Qualifying Test

1. What is the perimeter (in inches) of a square that has area 9 square inches?

    a. 8   b. 10   c. 12   d. 16

2. The graph of which of the following is perpendicular to the line $y + 3x = 7$?

    a. $y - 3x = 7$ b. $x - 3y = 7$ c. $3x - y = 7$ d. $x + 3y = 7$

3. Let T = $\sin\left(\cos\left(\pi/2\right)\right)$ (where $\pi/2$ is measured in radians). Then

    a. T = 0   b. $0 < T \leq 1/2$ c. $1/2 < T < 1$ d. T = 1

4. Sue rolls two fair six-sided dice (with faces numbered 1-6) and computes their sum, while Diane rolls a single fair dodecahedral die (with faces numbered 1-12). Which of the following numbers has the property that Sue and Diane are equally likely to roll that number?

    a. 8   b. 9   c. 10   d. 11

5. A positive integer is called *square-free* if it is not divisible by any perfect square greater than 1. Suppose m and n are square-free integers greater than 1, with $m \neq n$. Which of the following is not possible?

    a. *mn* is square-free and composite   b. *m/n* is prime   c. *m/n* is square-free and composite

    d. $\sqrt{mn}$ is rational

6. Let $f(x) = 5x^3(2x + 3)^4$ and $g(x) = 50x^5(8x - 4)^2$. Which of the following is closest to $f(10^6) \div g(10^6)$?

    a. 0.025   b. 0.25   c. 2.5   d. 25

7. The graph of the equation $(x + 2)^2 + (y - 3)^2 = 12$ contains points in all quadrants except quadrant

    a. I   b. II   c. III   d. IV

8. A,B,C,D,E,F,G, and H represent eight different digits selected from {1,2,…,9}. If (A + B)/(C + D) + (E + F)/(G + H) is as large as possible, which digit is not used?

    a. 1   b. 2   c. 4   d. 5

9. Suppose x is an integer satisfying $\log_3(9x) + \log_9(3x) = 7$. What is the ones digit of x?

    a. 3   b. 5   c. 7   d. 9

10. How many ordered pairs of positive integers (m, n) are there such that $m^2n^5 = 20^{20}$?

    a. 15   b. 20   c. 30   d. 40