2019 Who Wants to Be a Mathematician Round One Qualifying Test

1. What is the ones (units) digit of $2019^2 - 2018^2$?

   a. 1   b. 3   c. 5   d. 7

2. What is the area in the first quadrant bounded by the graph of $x + 2y = 4$ and the x- and y-axes?

   a. 1   b. 2   c. 4   d. 8

3. Let $f(x) = x^2 + 7$. What is $f\big(f(2)\big)$?

   a. 11   b. 107   c. 121   d. 128

4. The radius of a sphere is 6 cm. What is the sphere's volume divided by its surface area (ignore units)?

   a. 1.5   b. 2   c. 2.5   d. 3

5. Which of the following is the closest integer to the cube of $\tan(\pi/3)$? ($\pi/3$ is in radians, not degrees)

   a. 5   b. 6   c. 7   d. 8

6. Let n represent a positive integer greater than 1. The number of points of intersection of the graphs of $y = x^n$ and $y = n^x$ is

   a. always odd   b. always even   c. odd when n is even and even when n is odd

   d. even when n is even and odd when n is odd

7. On a flat surface, a bug walks 1 foot north, 2 feet west, 3 feet south, 4 feet east, 5 feet north, and 6 feet west. It then walks straight back to its original starting point. How far did the bug walk total, in feet?

   a. 21   b. 24   c. 26   d. $21 + \sqrt{5}$

8. Double the sum of the first 1,111 positive integers and subtract 1,111. What is the result?

   a. 616,605   b. 1,234,321   c. 1,423,231   d. 1,432,231

9. Suppose $\log_{10} 8 = r$ and $\log_{10} 9 = s$. What is $\log_{10} 5$ in terms of r and/or s?

   a. $\sqrt[3]{r} + \sqrt{s}$ b. $\big(\sqrt[3]{r}\big)\big(\sqrt{s}\big)$ c. $(r/3) + (s/2)$ d. $1 - (r/3)$

10. Which of the following is closest to the number of ordered pairs of points (m, n), where m and n are both between 1 and 100 inclusive and relatively prime (their greatest common divisor is 1)? [(2,3) and (3,2) count as two such points.]

    a. 5,000   b. 6,000   c. 7,000   d. 8,000