

## 2019 Who Wants to Be a Mathematician Round Two Qualifying Test

1. Find the remainder when  $2019^{2019} + 2020^{2019}$  is divided by 7. \_\_\_\_\_
2. If  $n$  is the fifth power of a positive integer, which of the following could be the number of positive integer divisors (or factors) of  $n$ ? (including 1 and  $n$ ) (Circle one of the choices below.)
- a. 451      b. 452      c. 453      d. 454      e. 455

3. How many solutions are there to  $\sin(4x) = \cos(2x)$  in  $[0, 2\pi]$ ? ( $x$  in radians) \_\_\_\_\_

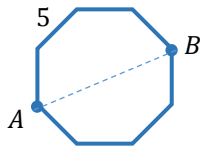
4. What is the next term in this geometric sequence:  $\sqrt{2}, \sqrt[3]{2}, \sqrt[6]{2}$ ? \_\_\_\_\_

5. If  $a$  is a positive real number, in how many points can the graphs of  $y = ax$  and  $y = x^2 + 1$  intersect? (Circle one of the choices below.)

- a. 0 only      b. 0 or 1 only      c. 0 or 2 only      d. 1 or 2 only      e. 0, 1, or 2

6. Express  $\sum_{n=1}^{10} \frac{2}{n(n+2)} - \sum_{n=1}^{10} \frac{1}{n(n+1)}$  as a fraction in lowest terms. \_\_\_\_\_

7. The regular octagon pictured has side length 5. What is the nearest integer to the length of AB? \_\_\_\_\_



8. The real polynomial  $p(x) = x^3 - x^2 + bx + c$  has  $2 + i$  as one of its roots. What is the sum of the  $x$ - and  $y$ -intercepts of the graph of  $y = p(x)$ ? (Your answer should not contain  $b$  or  $c$ .) \_\_\_\_\_

9. You roll two fair six-sided dice and your opponent rolls one die. What is the probability that the maximum of the numbers shown on the top faces of your two dice is strictly higher than the number shown on the top face of your opponent's die? \_\_\_\_\_

10. (tie-breaker) A *partition* of a positive integer  $n$  is a way of writing  $n$  as a sum of positive integers. For example, the number **3** has three partitions: 3 (the number itself is a partition with one addend),  $2 + 1$ , and  $1 + 1 + 1$  (the order of the addends doesn't matter). How many partitions does **101** have? \_\_\_\_\_