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CURRENT TRENDS IN GRADUATE EDUCATION IN PH. D. GRANTING MATHEMATICS DEPARTMENTS

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This article is a report of trends in graduate enrollments, financial aid, and related matters affecting Ph.D. programs in mathematics. It is based on recent numerical data, together with impressions obtained from departments through a questionnaire and other sources. We focus on mathematics departments in the U.S. which grant the Ph.D. A briefer report is given concerning other mathematical science departments granting the Ph.D., for which the data available were less complete, and for Canadian departments.

One of the most striking trends is the steady decline in mathematics graduate enrollments, from a peak which occurred around 1969. In the last three years alone, graduate enrollments in Ph.D. granting mathematics departments fell overall by 17% with a larger drop among certain departments (Table 1 below). The decline has been accompanied by a drop in numbers of applicants for graduate study and increased attrition from Ph.D. programs. Falling graduate enrollments may lead to a continuing decline in numbers of Ph.D. degrees, which would tend to improve job prospects for young Ph.D. mathematicians in the difficult job market expected during the early 1980s. On the other hand, the trend is producing stresses in many Ph.D. programs, particularly those for which the decline in graduate enrollments has been more severe than the nationwide average. Our capacity to produce Ph.D. mathematicians trained for traditional careers in university teaching and research exceeds the demand. There is need to achieve more diversity among graduate programs. For instance, some departments are reorienting their programs to better meet needs of industry and government, with emphasis on the master's degree.

The downward trend in graduate mathematics enrollments parallels that in the physical sciences. For mathematics and the physical sciences, there is strong evidence that future employment prospects, as perceived by students, are important considerations in decisions about pursuing graduate study. In contrast, graduate enrollments in humanities and social sciences remain strong despite job prospects, which are often bleak. Better employment prospects for applied mathematicians have stimulated an interest in applications among mathematics graduate students. Many continue toward the Ph.D. in pure mathematics while taking applied courses; others choose thesis topics in applied areas.

As regards financial support, the important change is the drastic decline since 1968 in U.S. Federal Government support of graduate students. Earlier fears that this might shift Ph.D. production from strong programs to weaker ones have proved unfounded.

In recent years total numbers of teaching assistants have not changed much nationwide, although individual departments have reported significant increases or decreases. Some departments now have serious difficulty finding enough qualified teaching assistants, and several have deliberately converted teaching assistantships into junior faculty positions (see the related article by Martha K. Smith, p. 366 of the November 1975 *Notices*).

Graduate enrollments, fall 1969–fall 1974. Table 1 shows total enrollments by full-time graduate students and by first-year students, in U.S. mathematics departments which grant the Ph.D. These departments have been divided into three groups: I, II, III, according to the American Council on Education ratings as explained on the following page. Table 1 is based on enrollment counts by department from the 1970, 1972, and 1975 editions of the Mathematical Association of America Guidebook. First year student enrollments in Table 1 may be slightly high, since some departments appear to have included part-time students in the data.

Full-time Graduate Enrollment in 155 Ph.D. Granting Mathematics Departments

Total Full-time	Fall 1969	Fall 1971	Fall 1974
Group I (Top 27 ACE Ranked)	3,060	2,760	2,490
Group II (38 other ACE Rated)	3,020	2,860	2,090
Group III (90 Unrated)	3,060	3,160	2,920
Total	9,140	8,780	7,500

First-year Full-time	Fall 1969	Fall 1971	Fall 1974
Group I	1,035	775	685
Group II	1,150	945	675
Group III	1,175	1,175	1,015
Total	3,350	2,895	2,375

Source: MAA Guidebook 1970, 1972, 1975.

TABLE 1

In this article Ph.D. granting departments in the mathematical sciences are classified as below. Groups I–V are departments in the U.S.

Group I: the top 27 ACE ranked mathematics departments
Group II: the other 38 ACE rated mathematics departments
Group III: 90 ACE unrated mathematics departments
Group IV: statistics, biostatistics and biometry departments
Group V: other mathematical science departments
Group VI: departments in the mathematical sciences in Canadian universities

For an account of the ACE ratings referred to above see "A Rating of Graduate Programs" by Kenneth D. Roose and Charles J. Andersen, American Council of Education, Washington, D.C., 1969, 115 pp. The information on mathematics was reprinted by the Society and can be found on pages 338–340 of the February 1971 issue of these *Notices*.

Table 1 shows an enrollment decline from fall 1969 to fall 1974 in all three Groups I–III, with an especially pronounced drop in Group II since fall 1971. Enrollment trends varied considerably from department to department. A few Group I and II departments reported modest increases in graduate enrollments. Over one quarter of Group III departments reported increases; often these were departments with rather new programs. However, these new programs are generally smaller and produce relatively few Ph.D.'s. Several departments in Groups II and III experienced a drop in graduate enrollments by half or more between fall 1969 and fall 1974. No significant difference was noted in overall graduate enrollment trends between mathematics departments in public vs. private universities.

The numbers in Table 1 were compared with AMS Survey data, and with NSF data on graduate enrollments included in Table 5 below. These sources confirm the trend shown in Table 1. It appears that the enrollment drop continued into the present academic year. AMS Survey data collected during summer 1975 show an estimated drop of about 2% in total full-time graduate student enrollments for Groups I–III, and of about 5% in first-year student enrollments, between fall 1974 and fall 1975. The actual decline for fall 1975 was probably greater, since corresponding estimates in previous AMS Surveys have subsequently turned out to be high.

Table 2 shows estimated enrollments for 157 other Ph.D.-granting departments. Not included in Table 2 are departments for which no graduate enrollment data appear in recent MAA Guidebooks. Some Group V departments are part of engineering divisions, and their mathematical science graduate enrollments are not always clearly separated. Moreover, for many departments included in Table 2, graduate enrollment data did not appear in each edition of the MAA Guidebook. Table 2 is less reliable than Table 1, both as an indicator of enrollment totals and trends. This is particularly true for Group V departments.

Estimated Full-time Graduate Enrollment in 157 other Ph.D.-Granting Mathematical Science Departments, and Canadian Departments.

Total Full-time	Fall 1969	Fall 1971	Fall 1974
Group IV (54 statistics related departments)	1,700	1,620	1,670
Group V (76 other mathematical science departments)	3,150	4,050	4,250
Group VI (27 Canadian departments)	1,260	1,290	1,110
First-year Full-time			
Group IV	510	530	580
Group V	1,450	1,530	1,530
Group VI	510	430	410

Source: MAA Guidebook 1970, 1972, 1975.

TABLE 2

In contrast to the departments in Table 1, Table 2 shows enrollment increases among the applications-oriented departments in Groups IV and V. AMS Survey data for the last three years show a similar trend. However, those data show larger increases in first-year graduate enrollments for Group IV (nearly 15% per year), and also in total full-time enrollments for Group V (about 7% per year). For Canadian departments, AMS Survey data show somewhat more pronounced recent enrollment declines than Table 2.

Trends in numbers of applicants, attrition, and shifts toward applied mathematics. In spring 1975 chairmen of departments in Groups I, II, III were asked for impressions of current trends in their departments. The purpose of this questionnaire was to supplement numerical data already available. Questions concerned trends in numbers and quality of applicants for admission to graduate study, attrition from Ph.D. programs, graduate student involvement with applied mathematics, and teaching assistantships. Responses were received from 125 of 155 departments currently in Groups I–III.

Applicants. 24% of the departments responding reported a significant decline during the last three years in numbers of applicants for admission to graduate study. Another 40% reported a slight decline, 27% reported the number of applicants as about the same, and 9% reported an increase. No discernable trend regarding the quality of applicants was found from responses to a question on this matter. The number of departments reporting that applicant quality had improved was about the same as the number reporting it had declined. Many reported little change in quality of applicants.

Attrition. Two questions were asked, one regarding attrition during the first two years of graduate study, the other about later attrition. Nearly 50% of the departments reported an increase in numbers of students leaving the department after two years or less of graduate study. However, fewer than 30% reported an increase in numbers of students past the second year leaving without the Ph. D. The discouraging job market was most often cited as a reason for increased attrition. Changes in the quality, attitudes, and objectives of graduate students were also considered important. A number of respondents cited toughened standards for the Ph. D. as a cause for increased attrition. However, decreased financial aid was generally reported as unimportant in graduate student attrition.

Student involvement with applied mathematics. Departments were asked about trends in (a) the number of students planning a Ph. D. thesis in pure mathematics, but taking applied courses; and (b) the number planning a Ph. D. thesis in applied mathematics.

Percentage of Group I–III Departments Reporting Indicated Change

Number of students who	Increasing significantly	Increasing slightly	No change or decreasing
(a) Plan a Ph. D. thesis in pure mathematics but take applied courses	42%	39%	19%
(b) Plan a Ph. D. thesis in applied mathematics	19%	44%	37%

TABLE 3

Table 3 shows that 81% of departments reported an increase in (a), and 63% an increase in (b). This trend can be at least partly attributed to students' perceptions of job prospects, which are currently better for people with applied skills. However, some students find a natural fit between a Ph. D. thesis topic in mathematics and an area of application. See an article by John Nohel, December 1975 *Notices*, p. 380.

The responses to (b) in Table 3 can be compared with recent data on Ph. D. production. The total number of mathematical science Ph. D.'s has recently remained fairly constant, in the range of 1,200–1,300 per year. However, the percentage of pure mathematics Ph. D.'s has declined to about half the total, with a corresponding rise in numbers of Ph. D. degrees classified as applied. In the academic year 1974–1975, Group I–III departments produced roughly 600 Ph. D.'s in pure mathematics and 170 in applied mathematics, besides those Ph. D.'s from Groups IV-VI. As reported by R. D. Anderson, November 1975 *Notices*, p. 359, there was a distinct shift toward Ph. D.'s in applied areas among Group II and III departments. Among departments reporting both years, there were among Groups II and III, 35 fewer Ph. D.'s in pure mathematics during 1974–1975 than during 1973–1974, and 33 more in applied mathematics. On the other hand, Group I produced 51 more Ph. D.'s in pure mathematice and 4 fewer in applied mathematics.

Discussion. It is interesting to compare trends in mathematics with other fields. Graduate School Adjustments to the "New Depression" in Higher Education, by David W. Breneman, National Board on Graduate Education Technical Report November 3, 1975, presents an interesting study of trends in a number of fields. The trend in mathematics is similar to that in the physical sciences. NSF data for fiscal years 1968–1973 reported in that study (see also Table 5 below) show nearly as great a percentage enrollment decline in mathematics as the widely publicized drop in physics graduate enrollments. In contrast, graduate enrollments and Ph. D. production have remained strong in the humanities and social sciences despite general dismal job prospects. For example, a Modern Language Association Survey showed that, of some 1,000 doctorates in English seeking employment for 1974–1975, no more than half had any realistic expectation of an academic job. On the other hand, such fields as economics and clinical psychology have not been experiencing serious employment problems for Ph. D.'s.

Mathematics students, like those in the physical sciences, tend to view graduate study as an investment in preparation for a lifetime career. Some will wish to pursue mathematics under any circumstances, through love of it. The most gifted of them should be encouraged to do so. Many other students have reacted to changed career prospects in one of several ways. Some have declined to enter graduate study in mathematics. Others have left graduate school short of the Ph. D. or switched out of mathematics. Still others have chosen to continue toward the Ph. D. in mathematics, but to broaden their employment possibilities either by gaining experience in applied mathematics or by other means.

It is the responsibility of the mathematics community, and mathematics departments in particular, to insure that students are told the best current estimates of employment prospects for mathematicians. In this connection see the letter from the President of the AMS, Lipman Bers, December 1975 *Notices*, Inside Front Cover. This letter was addressed to chairmen of Ph. D. granting mathematics departments at the request

of the AMS Council.

Mathematics faculties have greatly expanded in recent years, and have become stronger. Some younger departments have become as strong in certain areas as more prestigious ones. As a result, our capability to produce young mathematicians geared to traditional university careers in teaching and research well exceeds the demand. It is estimated that during the early 1980s there may be as few as 75 tenured openings per year among all 155 departments in Groups I, II, and III. This roughly equals the expected number of vacancies due to deaths and retirements, estimated to be during this period about 1 1/2 % per year of the approximately 5,000 faculty members in Group I, II, and III departments. Mathematics departments not granting the Ph. D. may have around 200 such openings per year during the same period. (See December 1975 *Notices*, pp. 377–380.) In this situation young mathematicians will need to be prepared, educationally and psychologically, for a variety of tasks. Some of these will be of the traditional sort, in mathematics departments or in laboratories devoted to long-term basic research. Many others will use their talents in nontraditional ways, for instance through industrial research related to shorter term company goals or in working with educationally disadvantaged groups.

In a period of declining graduate enrollments, it seems wasteful for all departments simply to retrench simultaneously, trying to maintain all present Ph. D. programs on a reduced scale. A reassessment of the scope and objectives of graduate programs in mathematics is in order. Departments have different strengths, both in terms of faculty and in terms of job markets where their graduates are successfully placed. These suggest directions in which the various programs might advantageously evolve. Some departments have chosen to emphasize master's degree programs oriented toward employment in industry or government. In a forthcoming issue of these *Notices*, a report is expected on the panel discussion "The changing role of the master's degree", held at the San Antonio AMS meeting, January 1976.

Graduate student financial support. Table 4 shows the percentage of full-time graduate students in Group I–VI departments with various types of support during fall 1974. Among mathematics departments, especially those in Groups II and III, teaching assistantships are currently by far the most important source of graduate student support.

Graduate Student Financial Support, Fall 1974,
in Ph. D. Granting Mathematical Science Departments

Group	Fellowship or Scholarship	Teaching Fellowship or Teaching Assistantship	Research Assistantship	Other
I	16%	51%	7%	26%
II	9	68	3	20
III	6	71	2	21
IV	21	30	22	27
V	12	24	24	40
VI	28	47	19	6

Source: MAA Guidebook.
The data in Tables 1, 2, and 4 were compiled from the MAA Guidebooks by Ernest Davis.

TABLE 4

Other data show trends in financial support. Table 5 is based on NSF data for 120 Ph. D. granting mathematics departments, tabulated by the staff of the Board on Graduate Education. Of these 120 departments, 24 belong to Group I, 38 to Group II, and 54 to Group III. Enrollment totals in Table 5 for Fiscal Years 1970 and 1972 are lower than the corresponding totals for fall 1969 and fall 1971 in Table 1, since fewer departments are included.

Full-Time Graduate Students in 120 Ph. D. Granting Mathematics Departments

Source of Support	Fiscal Year 1968	1970	1972	1973
Federal Government	2,564	2,144	1,331	903
Institution, state, or local government	4,420	4,570	4,560	4,553
Self, loans, and family	1,586	1,477	1,612	1,617
Other	388	302	267	211
TOTAL Full-time Enrollment	8,958	8,493	7,770	7,284

Source: Data from NSF Graduate Student Support Surveys.

TABLE 5

The striking feature in Table 5 is the drastic decline in U.S. Federal Government support for mathematics graduate students, through fellowships, traineeships, and research assistantships. The total number of students federally supported was down by 1,661 from FY 1968 to FY 1973. Of this total, the drop was 699 among Group I departments, 587 among Group II, and 375 among Group III. This counts, of course, only departments among the 120 included in Table 5. During this period there was a great decline in federally funded graduate fellowships and traineeships in all fields,

down from 51,500 in FY 1968 to 6,600 in FY 1974. The decline in Federal support for graduate students appears to have affected some kinds of departments more than others. For departments in some private universities, with high tuition and relatively few teaching assistantships, availability of financial support for graduate students has become a real constraint. Moreover, without federally funded fellowships and traineeships, lower ranked departments often find it more difficult to compete for top students with more prestigious departments.

In addition to kinds of Federal support presently remaining for graduate students, the U. S. Federal government may be expected to have a continuing interest in support for training related to selected areas of special national concern (e.g. energy).

Support from institutions' own funds, state, or local governments shown in Table 5, is mainly through teaching assistantships in the case of mathematics. Numbers of students with such support showed little change between FY 1968 and FY 1973. More recent AMS Survey data show that total numbers of teaching assistants have not changed significantly since FY 1973. However, there is considerable variation from department to department. About 30% of departments responding to the questionnaire mentioned above reported either a significant increase or significant decrease in numbers of teaching assistants. (Apparently the increases and decreases offset each other, so that nationwide totals remain about the same.) Higher undergraduate enrollments were frequently cited as a reason by those reporting increases in numbers of teaching assistants; budget cuts and fewer graduate students were cited by those reporting decreases. Sixteen departments were identified as having deliberately converted teaching assistantships into junior faculty positions. The numbers of assistantships converted ranged from 1 or 2 up to 25 in the case of one department.

At present departments are competing for teaching assistants from a diminishing nationwide pool of graduate students. As graduate enrollments continue to decline, more departments encounter shortages of well-qualified teaching assistants. There are at least three ways in which institutions can respond to the shortage. One is to make teaching assistantships more attractive to prospective students. If this strategy succeeds, it may be optimal for the individual department. However, if successfully pursued by many departments, more young people will be attracted into the teaching profession with no long-term prospects of remaining in it. There is expected to be difficulty placing each year, as mathematics teachers at any level, numbers of the magnitude of the number of first-year students shown in Table 1 as entering fall 1974. A second possibility is to make do with available teaching assistants, even those marginally qualified or unqualified. This does not make educational sense, when well qualified Ph.D.'s are available to teach. The third alternative is to convert teaching assistantships into faculty positions. As already mentioned, this has been done at several institutions. Sometimes the conversion was made with no increase in cost. In other cases, additional funds for conversion were found. The article by Martha K. Smith (November 1975 *Notices*), cited earlier, discusses this matter further.