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**Maxwell Bileschi\***, Department of Mathematics, University of Buffalo, 244 Mathematics Building, Buffalo, NY 14260-2900, and **Meridangela Gutierrez Jhong**, School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY 14623. *A complete characterization of optimal vertex rankings of paths, cycles, and joins of graphs.*

A  $k$ -ranking of a graph is a labeling of the vertices with  $\{1, 2, \dots, k\}$  where any path between two vertices of the same label contains a vertex with a strictly larger label. These rankings have been applied to the scheduling of manufacturing systems, monitoring of communication networks, factorization of Cholesky matrices in parallel, and VLSI layout generation.

Following along the lines of the chromatic number, the rank number of a graph,  $\chi_r(G)$ , is defined to be the smallest  $k$  such that  $G$  has a  $k$ -ranking. In addition to  $\chi_r(G)$ -rankings, we investigate sum-optimal rankings, where the sum over all labels is minimized. While  $k$ -rankings and sum-optimal rankings have been studied, little has been done to quantify the number of these rankings. We use tools from both graph theory and combinatorics to characterize and enumerate all possible  $\chi_r$ -rankings and sum-optimal rankings for paths, cycles, and joins of graphs. It is likely that the ideas presented in this project can be applied to larger families of graphs. (Received August 10, 2011)