How to Give a Good Colloquium*

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Most colloquia are bad. They are too technical, and aimed at too specialized an audience. Consequently, most mathematicians skip colloquia in areas not in their general field (unless the speaker is famous: mathematicians are very class-conscious). So when a conscientious speaker actually listens to the routinely ignored advice to prepare a lecture "accessible to graduate students", he or she looks out on the audience and sees only experts in the field, and feels stupid for preparing an elementary lecture.

But the colloquium should be the center of the department's week, the time when all the faculty and graduate students get together to learn of somebody else's perspective on mathematics, and to broaden their own. Many person-hours go into listening to colloquia; the speaker has an obligation to not waste them.

Here are some suggestions on giving a colloquium. They are guidelines, not absolute rules.

1. Don't be intimidated by the audience.

Just because John von Neumann is in the audience does not mean you have to aim the lecture at him. All too often one hears remarks like "I'm sure everybody here knows ...". What this really means is that the speaker knows 3 members of the audience in his or her specialty, and to avoid the risk that they find parts of the talk trivial, the speaker ignores the remainder of the audience. The speaker also incorrectly assumes that other members of the department must have at least a nodding acquaintance with their colleagues' work.

2. Don't try to impress the audience with your brilliance.

Making the talk complicated so that your work appears profound is a greater sin than being intimidated, because it stems from vanity rather than insecurity. The effect is just as bad: the speaker presupposes that the audience knows what a class number field, bornology and Koszul complex are, and loses everyone. Such a talk is often preceded by an "apology to the experts", just to make sure everyone realizes that this stuff is all trivial to the great mind at the blackboard.

3. The first 20 minutes should be completely understandable to graduate students.

Be honest when deciding what a graduate student knows. It does not mean that the first 20 minutes should be understandable to a student who knew everything you did just before you got your Ph.D., plus a few things you learned since then but feel you ought to have known; it means a student who just scraped by the required coursework in your area, and went into a different field. So you can assume they know what L^p is, but not what a Sobolov space or pseudo-differential operator is; you can assume they know what a manifold is, but not what Poincaré duality is; you can assume they know what a field extension is, but not what an induced representation is.

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4. Carry everyone along.

The rest of the talk, except perhaps for the last 5 minutes, should be understandable in outline, if not in every detail, by an alert (but not brilliant) student. This means giving informal descriptions along with (or instead of) formal definitions; explaining examples of what you are talking about; and periodically regrouping so that the audience can understand what the main idea is. Even the theorems do not have to be stated precisely: you can add the hypothesis that something be "nice" without saying exactly what "nice" means.

5. Talk about examples.

Choose an example that illustrates the main point of the theorem. If you have a theorem that applies to all strictly pseudo-convex domains, but is non-trivial on the ball, just talk about the ball. Don't strive for generality. And if there are no examples that you can explain in the course of the talk, then the theorem is probably not suitable for a colloquium talk.

6. Prove only tautologies.

Often the original definition of an object is not the way you want to think about it as your talk develops. It is useful to prove the (perhaps trivial) equivalence of two different ways of looking at something, so the audience can actually see the connection rather than having to take it on faith.

Proving a real theorem, though, is pointless; nobody will understand the proof, not even those in the audience whom you assume are experts.

7. Put the theorem in context.

Discuss the history of the problem — how it is connected to earlier results, and how it relates to the major problems in the field.

8. Pay attention to the audience.

When teaching calculus, you know (or should) that looking at students' faces is a good way of gauging how they are following the material, and whether you need to slow down, or to go back over what you have just done. Do the same when giving a colloquium (and just because one person in the front row is nodding in agreement does not mean that everyone is following).

I was once at a colloquium where, 3 minutes into it, a member of the audience asked "could you define a von Neumann algebra please?" The audience member, as the speaker knew, was an expert in von Neumann algebras. The question clearly meant "You are pitching this talk at too high a level — make it more elementary." Unfortunately, the speaker did not get the hint, gave a quick definition, and proceeded to give a very erudite talk that at most 2 of the 40 people in the audience could follow at all.

9. Don't introduce too many ideas.

I once went to a talk (given to an undergraduate audience), in which the speaker started out by defining a manifold, and 30 minutes later was talking about Chern classes. This was ridiculous. Even though he defined everything logically necessary to understand the definition, nobody who started out not knowing what a manifold was could have absorbed all the ideas necessary to understand what a Chern class is.

We know that, when teaching calculus, one cannot on the first day of class give the definition of a limit, derivative and integral and go on to prove the fundamental theorem of calculus. So, too, every member of the colloquium audience can only take in a couple of new concepts. If you introduce too many, the audience will cease to understand you.

10. Write an abstract.

The main purpose of the abstract is to advertise your talk and to attract people's interest. It should also indicate the level at which the talk will be pitched, and what prerequisites will be assumed. Be honest — if you say your talk is accessible to graduate students, make sure that it is.

11. Find out in advance how long the colloquium is, and prepare accordingly.

Some are 50 minutes, some are 60 minutes (ours are 57 minutes long, after the audience settles down and the introduction is made). It is all right to end 5 minutes early; it is not all right to end 5 minutes late.

12. Don't use an overhead projector.

It is possible to give a good talk with an overhead projector, but most people are better off without them.

Even if you can resist the temptation to go too fast, there is still the insuperable problem that material does not stay around long enough. During a blackboard talk, if I forget what T_1 is, or was not paying attention when it was defined, I can look at the other side of the board and be reminded. In a well-designed lecture hall with plenty of blackboard space, you should have to erase something only after it has been on the board for at least 30 minutes; the important things can stay unerased for the whole lecture.

If you have complicated pictures to show, then you may be forced to use an overhead projector. Try to set it up on one side, so that you still have plenty of blackboard space for the non-pictorial part of your talk.

13. You do not have to talk about your own work.

There is much to be said for talking about somebody else's work. Unless you happen to have recently revolutionized the field, your latest theorem is probably pretty technical, and by the time you explain everything necessary to understand the statement, you will have lost much of the audience. But if you are willing to talk about somebody else's work, you can (i) choose a more important theorem and (ii) more easily simplify it to get the big idea across (if you sweated for 6 months to get the extra epsilon in the theorem, you will want to include it; if somebody else sweated for 6 months, you can easily drop it).

It is, of course, rare for a colloquium speaker not to mention his or her own work, and it is perfectly legitimate to want to let the audience know what you have been doing. But you should give a broad overview of the field, and wait till the end to describe your latest contribution.

I end with some advice to audience members and colloquium organizers.

1. Don't be too polite.

I have heard terrible colloquia from senior mathematicians, who have been giving bad talks for 30 years. I can only conclude that they do not realize their talks are bad. Why? Because afterwards, people come up politely and say "Nice Talk", thinking it is a harmless white lie. It is not: it means that the next unfortunate audience will have to sit through a bad talk, the speaker obliviously thinking that he or she is doing a great job.

2. Subsidize graduate students who dine with the speaker.

An important part of the colloquium is the post-colloquium dinner, where people chat with the speaker about their departments, exchange gossip, and discuss mathematics. A relatively quiet restaurant that facilitates group discussions should be chosen. It is good for graduate students to come to these dinners —it is part of joining the mathematical community. Faculty should ensure that graduate students can afford to do so by subsidizing their share of the bill.

3. Consider asking the speaker to give two talks, a seminar and a colloquium.

In the seminar, the speaker can describe his or her latest theorems in detail, and what clever ideas went into the proof. At the colloquium there is then less incentive to try to impress the audience.

It is an honor to be invited to give a colloquium. It is not the occasion to tell a few of your friends the details of your work. It is your opportunity to show mathematicians in other areas and other fields the exciting work that is going on in your area. Seize this opportunity.

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